Outline

- We will move to room 210 from Wednesday (except Wed Oct 8 – room 170)
- Policy addition
- HW review
- RC circuits
  - Differentiation/integration of signal
  - Response to sine waves
  - High/low pass filters

HW grading

- Only overall grades are given.
- If HW is pretty much perfect, you get 100% (√⁺; not quite perfect; or √⁺⁺)
- If something substantial is desired, you get 80% (√⁻; or √⁻ if something significant is missing)
- If not submitted on time, or very little effort is made, you get 0%.

Tardiness to lab

- If you miss more than 5 minutes of lab (like being late), it will count as half of absence.
- If you miss more than 30 minutes of lab (like being late), it will count as an absence.
- Would copies of PP files on web useful?

HW Q6

- From (4), \( V_{\text{out}} = V_{\text{in}} \frac{R_2}{R_L} \frac{R_1}{R_1 + R_2 + R_3} \).
- From (5), \( V_{\text{out}} = V_e \frac{R_2}{R_e + R_1} \).
- So for (6), these two expression must give equal value for any values of \( R_e \).
- \( V_{\text{in}} \frac{R_2}{R_1 + R_2 + R_3} \frac{R_1}{R_1 + R_2 + R_3} = V_e \frac{R_2}{R_e + R_1} \).

Arithmetic

- \( V_{\text{in}} \frac{R_2}{R_2 + R_1} \frac{R_1}{R_1 + R_2 + R_3} (R_1 + R_2 + R_3) = V_{\text{in}} \frac{R_2}{R_1 + R_2 + R_3} \frac{R_1}{R_1 + R_2 + R_3} \).
- Simplify LHS (multiply num/den by \( R_2 + R_1 \):)
  - \( V_{\text{in}} \frac{R_2}{R_1 + R_2 + R_3} + R_1 = V_{\text{in}} \frac{R_2}{R_1 + R_2 + R_3} + R_1 
  - [V_{\text{in}} \frac{R_2}{R_1 + R_2 + R_3} + R_1] = V_{\text{in}} \frac{R_2}{R_1 + R_2 + R_3} + R_1 
  - [V_{\text{in}} \frac{R_2}{R_1 + R_2 + R_3} + R_1] = V_{\text{in}} \frac{R_2}{R_1 + R_2 + R_3} + R_1 
  - V_{\text{in}} \frac{R_2}{R_1 + R_2 + R_3} = R_1 = R_1 \frac{R_2}{R_1 + R_2 + R_3} 

Arithmetic II

- Or from
  - \( V_{\text{in}} \frac{R_2}{R_2 + R_1} \frac{R_1}{R_1 + R_2 + R_3} = V_{\text{in}} \frac{R_2}{R_2 + R_1} \frac{R_1}{R_1 + R_2 + R_3} \).
  - Remove denominators
  - \( V_{\text{in}} \frac{R_2}{R_2 + R_1} \frac{R_1}{R_1 + R_2 + R_3} = V_{\text{in}} \frac{R_2}{R_2 + R_1} \frac{R_1}{R_1 + R_2 + R_3} \).
  - Eliminate \( R_1 \) and equate terms with the same power of \( R_2 \).
  - \( V_{\text{in}} \frac{R_2}{R_2 + R_1} \frac{R_1}{R_1 + R_2 + R_3} = V_{\text{in}} \frac{R_2}{R_2 + R_1} \frac{R_1}{R_1 + R_2 + R_3} \).
  - \( V_{\text{in}} \frac{R_2}{R_2 + R_1} \frac{R_1}{R_1 + R_2 + R_3} = V_{\text{in}} \frac{R_2}{R_2 + R_1} \frac{R_1}{R_1 + R_2 + R_3} \).
HW Q9

• From (7),
  – \( V_{\text{out}} = 110 \times R_2 / (R_1 + R_2) \) for (b) and
  – \( V_{\text{out}} = V_e \) for (c).
  – For the two circuits to be equivalent there two
    must equal. So
  – \( 110 \times R_2 / (R_1 + R_2) = V_e \); same as (6).

HW Q9 Cont’d

• From (8),
  – \( I_s = 110 / R_1 \) for (b) and
  – \( I_s = V_e / R_e \) for (c).
  – For the two circuits to be equivalent there two
    must equal. So
  – \( 110 / R_1 = V_e / R_e \).
  – So, \( R_e = V_e \times R_1 / 110 = 110 \times R_2 / (R_1 + R_2) \times R_1 / 110 \)
    \( = R_1 R_2 / (R_1 + R_2) \); same as (6).

Recap

• \( I = C \frac{dV}{dt} \): Ohm’s Law for capacitor.
• Kirchhoff’s Laws still apply to circuits, and
everything is the same.
• Now everything is a function of time, and
• \( dV/dt \) or \( \int I \, dt \) is involved, so equations
  will be differential equations, rather than
  algebra equations.

Simple RC circuit(s)

• General solution is \( V_1 = A e^{-t/RC} \), and
• Specific solution is \( V_1 = \frac{\exp(-t/RC) \int_0^t e^{(t-\tau)/RC} V_0(\tau) \, d\tau}{RC} \)

Specific case I

• \( V_1(0) = 0 \) for \( t < 0 \) and
  – \( V_1 = V_0 \) for \( t > 0 \). i.e. step function input.
• Then
  – \( V_1(0) = V_0 - A e^{-0} \).
  – Assuming that \( V_1(0) = 0, A = V_0 \).
  – \( V_1 = V_0 (1 - e^{-t/RC}) \).
• One may not intentionally
  soften the wedge of a square pulse
• Unwanted capacitance in a
circuit may do this to distort pulses.

Simple RC circuit II

• Just switched \( R \) and \( C \).
Just “reverse”

- \( V_2(t) = V_m(t) - V_1(t) \).
- For step function input voltage, \( V_2(t) = V_0 e^{-t/RC} \).
- This is useful when you want to pick up changing part of your signals.
- Examples?

Differentiator

- This same circuit is also called “differentiator.”
- For example, if the input is constantly rising voltage:
  \( V_2(t) = aRC \times (1 - e^{-t/RC}) \).
- Does not look exactly like the constant you would expect from name.

- \( V_2(t) = \alpha C \times (1 - e^{-t/RC}) \).
- However, it is \( \sim \alpha RC \)
  when \( t = RC \), which is proportional to \( \alpha = \frac{dV_1}{dt} \).

So is this an integrator?

- Yes.
- For the step function input, for example,
  \( V_2(t) = V_0(1 - e^{-t/RC}) \).
  When \( t = RC \),
  \( V_2(t) = V_0(tRC) \),
  which increases at a constant rate.

How do these circuits behave with sine-shaped input?

- \( V_m(t) = A \cos(\omega t) \)
- Assume that \( V_1 = B \cos(\omega t + \varphi) \)
- Substitute this into \( (V_m - V_1)/R = C \frac{dV_1}{dt} \)
- \( A \cos(\omega t) - B \cos(\omega t + \varphi) = -RC\omega B \sin(\omega t + \varphi) \)
- \( [A - B \cos\varphi] \cos\omega t + B \sin\varphi \sin\omega t = -RC\omega B [\sin\varphi \cos\omega t + \cos\varphi \sin\omega t] \)

More math!

- \( A - B \cos\varphi = -RC\omega B \sin\varphi \)
- \( B \sin\varphi = -RC\omega B \cos\varphi \)
- From the 2nd Eq. \( \tan\varphi = -RC\omega \), or \( \varphi = -\tan^{-1}(RC\omega) \).
- From the first Eq. \( B = A/[\cos\varphi - RC\omega \sin\varphi] = A/[1+(RC\omega)^2]^{1/2} \).
What does this mean?

- \( \varphi = -\tan^{-1}(RC\omega) \),
- \( B = A[1+(RC\omega)^2]^{1/2} \).

- The first one says that the output signal is shifted in phase relative to the input.
- The 2nd line says the voltage divider works a bit different from resistive voltage divider!

When \( \omega \) is \( \sim 0 \), or \( \rightarrow \infty \)

- \( \omega \rightarrow 0 \)
  - \( \varphi = -\tan^{-1}(RC\omega) \rightarrow 0 \), and \( B = A \). i.e. no change in signal

- \( \omega \rightarrow \infty (RC\omega \gg 1) \)
  - \( \varphi \rightarrow -\pi/2 (-90^\circ) \) and \( B \ll A \)
    - If cosine is input, output is (negative sign here I had in the lecture was wrong) sine, like integration.
    - Considering that \( t \propto 1/\omega \), this is the condition as \( t \ll RC \).