Outline

- Measurements, errors accuracy and precision
  - Distributions (Poisson)
  - Least Square Fitting

Poisson distribution

- If you observe nuclear decays for time $T$, if $N$ the number of radioactive isotopes (atoms), each of them can decay, so the number you would observe the Binomial distribution. However, in this case, $N$ is very large (of the order $N_{\text{avogadro}}$) and $p$ is very small, but their product $Np$ is reasonable so that you will observe finite number of decays.
- In this case, if you use the Binomial distribution, which is a correct one of use, computation will be enormous.
- When you realize that in the Binomial distribution, $\text{Prob}(n+1) = \text{Prob}(n) * p * (N-n)/(n+1)$, in the extreme that $N \to \infty$ and $p \to 0$, but $Np = \mu$ (average decays), then $\text{Prob}(n+1) = \text{Prob}(n) * \mu / (n+1)$. Using this recursive relation and noting that the sum of $\text{Prob}(n)$ is one, one can figure out $\text{Prob}(n) = e^{-\mu} \frac{\mu^n}{n!}$. [note that if you expand $e^{\mu} = 1 + \mu + \frac{\mu^2}{2!} + \frac{\mu^3}{3!} + \ldots$ $\mu^n/n! + \ldots$ These terms satisfy the above recursive relation. So you have to normalize so that their sum is one – that’s why you have $e^{-\mu}$]
- Almost by definition, the average must be $\mu$. And it is because:
  - $\langle n \rangle = \frac{(0*1+1*\mu+2*\mu^2/2!+3*\mu^3/3!+\ldots n*\mu^n/n!+\ldots )/(1+\mu+\mu^2/2!+\mu^3/3!+\ldots \mu^n/n!+\ldots )}{(1+\mu+\mu^2/2!+\mu^3/3!+\ldots \mu^n/n!+\ldots )}$.
  - Since $e^{\alpha \mu} = 1+\alpha \mu + (\alpha \mu)^2/2!+ (\alpha \mu)^3/3!+ \ldots (\alpha \mu)^n/n!+\ldots$ then $d(e^{\alpha \mu})/d\alpha = 0*1+1*\mu+2*\alpha \mu^2/2!+3*\alpha^2 \mu^3/3!+\ldots n*\alpha^{n-1} \mu^n/n!+\ldots$ the denominator of the above expression is $e^{\alpha \mu}|_{\alpha=1}$ and the numerator is $d(e^{\alpha \mu})/d\alpha|_{\alpha=1}$.
  - Once you realize that $d(e^{\alpha \mu})/d\alpha = \mu e^{\alpha \mu}$ it is straightforward to find that $\langle n \rangle = \mu$.
- Similar trick can be used to show that $\langle n^2 \rangle = \mu + \mu^2$.
- Since $\sigma_n^2 = \langle n^2 \rangle - \langle n \rangle^2 = \mu$, or $\sigma_n = \sqrt{\mu}$.
- How large could $\langle n \rangle$ be if you observed $n = 0$ for a particle search if you can tolerate the “Wrongly (false) rejection” or Type I error probability of 10%? How about if the tolerated probability is 1%?