Resister circuits, $Z_{in}/Z_{out}$.
RC circuits

Recap:
- Using Genecon as an example of transducer (converting rotation to electrical signal), we showed that extracting the maximum power out of it by connecting a load optimum for power transfer will not be a good idea. The turning becomes too difficult. This is in general true. If the measuring circuit draws more power, it will disturb the system being measured more and its behavior changes.
- Genecon as another “battery” – the source of $Z_{out}$ is not (purely) the resistance of the wire inside it. In fact, the major contribution comes from the fact that when a low resistance load is connected, the turning becomes harder and gets slower. This causes the lowering of the output voltage. This effect can be well emulated by $Z_{out}$ and a fixed source voltage.
- In a dry battery, the fact that the speed of chemical reaction to supply charges to outside is limited causes lowering of the output voltage. Again, this effect can be well emulated by $Z_{out}$ and a fixed source voltage.

In the voltage divider circuit to produce 5 V from 9 V, the resistors are $R_1 = 10\,\text{kohm}$ and $R_2 = 12.5\,\text{kohm}$, the power consumption will be reduced to 4 mW, which seems a lot more reasonable than 4W in the previous strategy.

Now what would happen when the load is connected? I said the load draws 10-100 mA. Let’s say it draws 10 mA. Does anything happen to the voltage? How can we estimate the voltage in this condition?

Example solution:
Let’s define $I_1 =$ current through $R_1$, and $I_2 =$ current through $R_2$.
Then $I_1 = I_2 + 10\,\text{mA}$.
Also, $I_1 R_1 + I_2 R_2 = 9\,\text{V}$. Now we have 2 equations for 2 unknowns. So let’s solve.
Substitute the first into 2nd.
$(I_2 + 10\,\text{mA})R_1 + I_2 R_2 = 9\,\text{V}$; $I_2 (R_1 + R_2) + R_1 10\,\text{mA} = 9\,\text{V}$;
$I_2 = (9\,\text{V} - R_1 10\,\text{mA})/ (R_1 + R_2) = -91\,\text{V}/22.5\,\text{kohm} = -4\,\text{mA}$;
$V = I_2 R_2 = -4\,\text{mA}*12.5\,\text{kohm} = -50\,\text{V}$.

Does it make sense? No! In reality, the voltage will go down substantially (close to zero), so that the circuit will not consume 10 mA. Less. Then it won’t function.

So this voltage divider works fine until you connect the load.

What do we need to do to reduce this effect? Increase or decrease the value of $R_1$ and $R_2$, keeping their ratio constant?
HW: If you maintain at least 4.5 V when 10 mA is stolen by the load, what is the maximum value of \( R_1 \) and \( R_2 \)?

Can we make these calculation easier by introducing \( Z_{\text{out}} \)?

What is \( V \) and \( Z_{\text{out}} \) of this voltage divider?

If the voltage divider circuit is equivalent to the standard output circuit, for any value of \( R_{\text{load}} \), the voltage over the load should be equal.

For the divider circuit, the voltage as a function of \( R_{\text{load}} \) is:

\[
9V \times \frac{R_2}{\left(R_2 + R_{\text{load}}\right)}\left[\frac{1}{R_1 + \left(R_2 + R_{\text{load}}\right)}\right],
\]

where \( \left(R_2 + R_{\text{load}}\right) \) means the value of resistance for \( R_2 \) and \( R_{\text{load}} \) in parallel. This can be somewhat simplified to:

\[
9V \times \frac{R_2 R_{\text{load}}}{\left(R_2 + R_{\text{load}}\right) \left(R_1 + R_{\text{load}}\right)}.
\]

For the standard circuit, the voltage will be:

\[
VR_{\text{load}} / (R_{\text{load}} + Z_{\text{out}}).
\]

How can these two expressions be equivalent?

If we assume that they are:

\[
9V \times \frac{R_2 R_{\text{load}}}{\left(R_2 + R_{\text{load}}\right) \left(R_1 + R_{\text{load}}\right)} = VR_{\text{load}} / (R_{\text{load}} + Z_{\text{out}}).
\]

Eliminating the fractions,

\[
9V \times \frac{R_2 R_{\text{load}}}{R_{\text{load}} + Z_{\text{out}}} = VR_{\text{load}} \left[R_1 + R_{\text{load}} + R_2 R_{\text{load}}\right].
\]

Cancel \( R_{\text{load}} \),

\[
9V R_2 \left(R_{\text{load}} + Z_{\text{out}}\right) = VR_{\text{load}} \left[R_1 + R_{\text{load}} + R_2 R_{\text{load}}\right].
\]

If this equation were to be identity, term containing \( R_{\text{load}} \) and not containing it must be separately equal:

\[
9V R_2 R_{\text{load}} = VR_{\text{load}} R_1 R_2 + VR_{\text{load}} R_2 R_{\text{load}}, \quad \text{or} \quad 9V R_2 = VR_{\text{load}} R_1 + VR_{\text{load}} R_2,
\]

\[
9V R_2 Z_{\text{out}} = VR_1 R_2.
\]

From the 1st Eq. \( V = 9V R_2 / [R_1 + R_2] \), which we sort of knew, and

From the latter, \( 9V Z_{\text{out}} = VR_1 \), or \( Z_{\text{out}} = VR_1 / 9V = R_1 / 9V * R_2 / [R_1 + R_2] = R_1 R_2 / [R_1 + R_2] \). \( \text{i.e.} \ Z_{\text{out}} \) is the same as the parallel resistance of \( R_1 \) and \( R_2 \).
Once we know that, obviously, if we want the voltage to decrease no more than 0.5 V (10%) when the load of 5 V/10 mA = 500 ohm is connected, $Z_{\text{out}}$ must be at most 10% of 500 ohms, which is about 50 ohms.

From this, what values do you get for $R_1$ and $R_2$?

**RC circuits**

This part has not been typed up, but most of the contents are in the lecture note of 9/17 as “recap”.