1) (7 Points) One billion Chinese people were asked to make their best guess at the height of their emperor because the science adviser to the emperor said that the average of these guesses is one of the most precise estimate of the emperor’s height. Make reasonable and appropriate assumptions and estimate the error on this average. (5 points) After you obtain the error estimate, discuss if this error sounds right to you, and if there are any sources of error that your calculation does not take into account. (2 points)

2) (10 Points) When you were listening to your favorite music on your MP3 player, you heard a short burst of electric guitar sound of about 400 Hz, which you estimated lasted for about 0.1 second. How large a frequency range around 400 Hz are the Fourier transforms of the sound (encoded in the MP3 file) significantly different from zero? (5 points) Assuming that the sampling rate of the sound digitization is 40 kHz, how many Fourier transforms are in this frequency range? (Assume that the guitar sound only contains the fundamental and no harmonics.) (5 points)

3) (8 Points) In a measurement of light absorption in a semiconductor, one may detect the transmitted light using a photodiode, which produces DC current proportional to the detected light. Since the current is usually very small, one needs to amplify it and convert to a voltage signal. This can be done with an op-amp with a feedback resistor. However, in the current case, the amount of light absorbed increases exponentially over a very small wavelength range. It is sometimes necessary to measure a signal that varies over four or five orders of magnitude. This is very difficult to do with an ordinary amplifier. (either large signals result in saturated output, or small signals will be below the noise level. In this problem, we consider the following solution to this problem. PD is a photodiode to detect transmitted light. Instead of a resistor in the feedback loop, there is a diode D. The relation between the voltage across the diode and the current through it is given by \( I = I_0(e^{\alpha V} - 1) \), which would replace the usual Ohm’s Law for a resistor feedback. This circuit produces interesting voltage output as a function of the input current produced by PD. Figure out this relation between the input current, \( I_0 \), and the output voltage \( V_{\text{out}} \) in terms of \( I_0 \) and \( \alpha \).

[Diagram of the circuit with labels PD, D, IS, and VOut]
You can assume that the op-amp is ideal, i.e. the Golden rule applies. The photo-diode, PD, produces a current I, that is proportional to the intensity of the incident light.

Once you obtain the answer to the main question, explain why this circuit is a good choice for measuring light signals that vary over many orders of magnitude in a few sentences.

4) (7 Points)
To prove the existence (or lack thereof) of the elusive Loch Ness Monster, Nessie, the employees at Yeti, Inc employ an ultra high resolution 1 Gigapixel black and white camera with an 8 bit resolution that is connected to a computer over a wireless network. At random time intervals the camera takes a pair of snapshots, A and B, of the lake surface; these two snapshots, A and B, are separated by a short time interval. For all practical purposes, you may assume that the time interval between the first snapshot, A, and the second one, B, is so short that the two images would be exactly identical if the monster did not appear or move. On the other hand, if the monster (or anything else) appeared, disappeared or changed position between two snapshots, part of the image of A would differ from B.

With the huge amount of data collected by each snapshot, it is not feasible to upload the data from the camera into the computer through the wireless network and then to compare pixel by pixel to see if the two pictures differ. Instead, the camera has an internal data buffer large enough to hold the three images A, B, and Q. In addition, it contains a microprocessor that will perform simple Boolean logic operations on each pixel, i.e., it will AND, OR, NAND, NOR and XOR each pixel* of A with its corresponding pixel of B and then store the result in the result buffer, Q. (Specifically, the Boolean logic operation will be performed between the pixels corresponding to the same picture location, i.e., between the jth pixel of A, Aj, and the jth pixel of B, Bj and the result will be stored in Qj.)

Determine which logic operation(s) needs to be performed so that the following two conditions are met:
- the result buffer Qi will be all zeros if all the corresponding pixels in Ai and Bi were identical, i.e., the “no-monster” case;
- the result buffer Qi will contain non-zero values when the corresponding pixels in Ai and Bi were not identical, i.e., it will contain a picture (or the outline) of the “monster” or of anything that changed position between the two snapshots.

Express your answer in terms of typical Boolean logic notation using Aj, Bj, and Qj where j (0 < j < 10^9) denotes the pixel’s location.

*Note:
Since each pixel can take on an integer value between 0 and 255 (with 0 being black, 255 being white and any value in-between a different shade of gray) each pixel will consist of one byte of data, i.e., 8 bits. The Boolean operations are applied “bitwise” to the entire byte. For example, a Boolean operation on two corresponding pixels in A and B, Aj and Bj, will result in the least significant bit of Aj and Bj operated on and the result is stored in the least significant bit of Qj; next the second least significant bit of Aj and Bj are operated on and the result is stored in the second least significant bit of Qj and so on until all 8 bits have been operated on by the same operation. This is identical to the way the C-bitwise logic operators work.
5) (8 Points)
The circuit in Figure 2 is a variant of a synchronous counter circuit. Unlike your typical counter that starts at 0 and then counts up or down, this counter, if properly seeded, continuously repeats a number sequence that does not include the number 0. Note: Q2 is the most significant bit (MSB) and Q0 is the least significant bit, i.e., the LSB.

![Figure 2](image)

a) Determine the (numerical) values of the sequence if the counter starts with the numerical value 4, i.e., Q2 = 1, Q1 = 0, Q0 = 0. (Hint: A timing diagram might be helpful. Also, for partial credits, write down the three Boolean expressions for D2, D1 and D0 in terms of Q0, Q1 and Q2.) (4 points)

b) From part a) determine what happens if the counter is seeded with any value that is not part of its sequence. (From the introduction, you should at least know one such value.) (Note: you only need to determine the result for one such value which is not a part of the counting sequence; the result is identical for all other values that are not part of the counting sequence.) (4 points)

6) (10 Points)
Write a complete ANSI C console* function that calculates the GPA for a number of courses taken.

Your function must have three arguments. The first argument points to an array containing the number of credits for each course taken; the number of credits are stored as integers. The second argument points to an array that lists the corresponding grade for each course taken; the grades are already entered as a number of type double and do not need to be converted from a letter grade. The last argument contains the total number of courses taken and is an integer. Once your function has calculated the GPA it will return the result through the function itself, i.e., with a return statement.

Note: Do not write a complete C-program; only provide the function and its prototype that will fulfill above requirements. You may call your function anything except “main.” You are not allowed to use or rely on global variables in your function though you may use as many or as few additional local variables as you consider necessary. In your function, do not include any input or output statements such as `scanf` or `printf` or any include files. You will be graded on program logic and syntax mistakes.

Console* means you must not include a LabWindows Graphical User Interface (GUI) or any (callback) function for it.