Problem 1: (9.5 Points)
Consider the filter on the right:

a. Write an expression for the transfer function of this circuit.
\[ H(j\omega) = \frac{jL\omega}{jL\omega + R(1-LC\omega^2)} \]

b. Discuss the asymptotic behavior of this filter, at high and low frequency. What type of filter is it? Find the characteristic frequency of the filter.
\[ \omega \to 0: H \sim 0 \]
\[ \omega \to \infty: H \sim 0 \]
It is a band-pass filter.
\[ H = 1 \text{ when } (1-LC\omega^2) = 0 \Rightarrow \omega_c = 1/(LC)^{1/2} \]

c. What phase shift is introduced by this filter at very small frequencies, very large frequencies and at the characteristic frequency?
\[ \omega \to 0: H \sim jL\omega / R \Rightarrow \theta(j\omega) = \pi/2 \]
\[ \omega \to \infty: H \sim -j/(RC\omega) \Rightarrow \theta(j\omega) = -\pi/2 \]
\[ \omega = \omega_c \Rightarrow H \sim 1 \Rightarrow \theta(j\omega) = 0 \]

d. On a log-log scale, sketch the magnitude of the transfer function and the phase shift as a function of \( \omega \).
Problem 2: (2.0 Points)
Consider the diode circuit in Figure 1. The diodes are semi-ideal with a voltage threshold of 0.7 Volt. Determine the voltage at point A, \( V_A \).

Assume all diodes off, no current flow, diode drops are 10, 20, diode 2 must be on.

Diode 1 off, Diode 2 on:
\[ 20 = 5k \times i + 0.7V + 10k \times i \]
\[ i = 1.3mA \rightarrow \text{diode drop 1} = 10 - 5k \times 1.3 = 3.5 \]
\[ \Rightarrow \text{diode 1 must be on} \]

Both diodes are on:
\[ i(5k) = (10 - 0.7) / 5k = 1.86mA \]
\[ i(10k) = (0.7 - 0.7 - (-10)) / 10k = 1.0mA \]
\[ i(d1) = 1.86mA - 1.0mA = .86mA \]

\[ V_A = 1mA \times 10k - 10 = 0 \text{ Volt.} \]

Problem 3: (6.0 Points)
The circuit in Figure 3 uses a three terminal adjustable resistor, also know as a trim pot, familiar to you from the lab. The trim pot is used to obtain an adjustable output voltage, \( V_{out} \).

The resistance at the wiper terminal, pin 2, with respect to pin 1 is:
\[ R_{12} = \alpha R_o \quad (1) \]

The resistance at the wiper terminal, pin 2, with respect to pin 3 is:
\[ R_{23} = (1 - \alpha) R_o \quad (2) \]

Where:
• $R_o$ is the resistance between pins 1 and 3 and in this case it is 1 k;
• $\alpha$ is proportional to the angle that the trim pot has been adjusted and its range is: $0 \leq \alpha \leq 1$.

a) Without any load attached, what is the range of the output voltage?
   With no load: $-3V < V_{out} < 7V$

b) Calculate the Thevenin Resistance, $R_{Th}$, for the circuit at $V_{out}$ with respect to ground. (No load attached.)
   $R_{Th}$ can be found by shorting all V-sources; in this case $R_{Th}$ is just $R_{12}$ in parallel with $R_{23}$:
   $$R_{Th} = (\alpha (1 - \alpha)) R_o$$

c) Find an expression for $V_{out}$ with respect to ground in terms of $R_o$ and $\alpha$ when no load has been attached. (Hint: check your answer with the one obtained in a.)
   Current through $R_o$:
   $$\frac{7V - V_{out}}{R_{12}} = \frac{V_{out} - (-3V)}{R_{23}}$$
   solve for $V_{out}$:
   $$V_{out} = 7(1 - \alpha) - 3\alpha$$
   (Check answer $\alpha = 0$, $V_{out} = 7V$, $\alpha = 1$, $V_{out} = -3V$)

d) Find an expression for $V_{out}$ with respect to ground in terms of $R_o$ and $\alpha$ with a load $R_L$ attached between $V_{out}$ and ground.
   Since answer c) is the same as the Thevenin voltage, $V_{Th}$, $V_{out}$ will just be the voltage divider formed by $R_{Th}$ and $R_{load}$:
   $$V_{out} = \frac{V_{th} R_{Load}}{(R_{Th} + R_{Load})}$$
   $$= \frac{(7(1 - \alpha) - 3\alpha) R_{Load}}{(R_{Load} + (\alpha (1 - \alpha)) R_o)}$$

Problem 4: (7.5 Points)

The op-amps circuits below all use ideal op-amps powered by $Vcc = +12V$ and $VEE = -12V$. (Ideal means: infinite input impedance, infinite open loop gain, very low output impedance.)

Calculate $V_{out}$ for each circuit with $V_{in} = +3V$ for all circuits.
Figure 4a: $V_{out} = +12V$ (it’s a comparator, $\Delta V_{in} > 0$)
Figure 4b: $V_{out} = +12V$ (it’s a comparator, $\Delta V_{in} > 0$)
Figure 4c: $V_{out} = +9V$ (it’s a non-inverting amp with gain 1+2/1)
Figure 4d: $V_{out} = -12V$ (it’s a positive feedback, comparator with hysteresis; the switching voltage is $\pm (12/11)V$. Since the input exceeds both of these, and since $\Delta V_{in} < 0$, $V_{out} = -12 V$)
Figure 4e: $V_{out} = -12V$ (it’s an inverter that has been “saturated.”)