6.1. Introduction

The Faraday Effect describes a magneto-optical phenomenon, i.e., the interaction of light and magnetic field. It manifests itself in certain materials as the rotation of the plane of polarization of a light beam when a magnetic field is applied perpendicular to it.

A linearly polarized light beam is traveling in the direction of the magnetic field, $B_x$, through an isotropic dielectric medium. The applied magnetic field causes a rotation in the light's plane of polarization. The rotation angle, $\phi$, is proportional to the magnetic field strength, $B_x$, the length $L$ of the material in the field and its Verdet constant, $\nu$: If the magnetic field strength remains constant throughout the sample then the rotation in polarization angle can be expressed by:

$$\Phi = \nu B_x L$$

Figure 6.1. The Faraday Effect.
The Verdet constant itself depends on the medium, its temperature and the frequency of the polarized light. (Note: the conventional symbol for the Verdet constant is: $V$. To avoid confusion with voltage readings, we switched the symbol to the Greek letter $\nu$.)

Examples of practical applications of the Faraday Effect are optical modulators and optical switches, i.e., optical isolators. These devices allow electrical signals to control the light intensity. This is achieved by controlling the strength of an applied magnetic field, usually produced by a solenoid, surrounding the dielectric medium. By adding an optical polarizer after the medium, the intensity of the emerging light will be altered according to Malus law, see equation 6.5. Other applications involve the remote sensing of magnetic fields by monitoring the rotation angle of the polarized light emerging from a fiber optical cable whose path intercepts the magnetic field.

In this experiment, you will measure the Verdet constant for distilled water. It is an extremely small value. It is only $2/10000$ of a degree / Gauss cm! Even when we apply our maximum magnetic field of about 150 Gauss to the roughly 10 cm long water cell, we don’t expect to observe more than a fraction of a degree change in the polarization angle. Therefore, some advanced methods of experimental physics must be applied to observe such a small signal. It is the purpose of this experiment to familiarize you with noise canceling through filtering and averaging by using a lock-in detector and an (optical) bridge technique.

### 6.2. Experimental Methods

In this experiment, we determine $\nu$ by measuring the polarization angle, $\Phi$, as a function of the applied B-field. Therefore, we need to calibrate the B-field as a function of current and also position.

#### Calibration of the B-field as a Function of Current and Position

Equation 6.1. is only valid when the B-field throughout the sample remains constant. In our case, the B-field along the horizontal axis of the solenoid is a function both of the current flowing through the solenoid and (to a much smaller degree – due to edge effects) its position, i.e., $B_x = B_x(I, x)$. For such a case, the B-field needs to be integrated over the entire sample and equation 6.1.

Changes to:

$$\Phi = \nu \int_{-L}^{L} B_x(I, x) dx$$  \hspace{1cm} (6.1.a.)

From Ampere’s law it follows that the B-field inside a solenoid is linearly related to its current. So if we calibrate the B-field for some arbitrary current $I_o$, then we should be able to predict the B-field for any current $I$ using the following linear relationship:

$$B_x(I, x) = \frac{I}{I_o} B_x(I_o, x)$$  \hspace{1cm} (6.2.)

Combining these two equations results in the following equation that relates the change in polarization angle to an applied solenoid current $I$:

$$\Phi = \nu \int_{-L}^{L} B_x(I, x) dx = \nu \int_{-L}^{L} B_x(I_o, x) dx$$  \hspace{1cm} (6.3.a.)

The term $\frac{1}{I_o} \int_{-L}^{L} B_x(I_o, x) dx$ depends entirely on the geometry of the solenoid and the number of its windings. Since we will use the same solenoid throughout the experiment this term represents a constant yet to be determined. Arbitrarily, we denote it with the Greek variable $\Gamma$: 

65
\[ \Gamma \equiv \frac{1}{I_0} \int_{-\frac{x}{2}}^{\frac{x}{2}} B_x(I_0, x) \, dx \]  

(6.3.b.)

Using this notation reduces equation 6.3.a. to:

\[ \Phi = \nu \Gamma I \]  

(6.3.c.)

From this equation we see that determining Verdet’s constant requires the following steps: First we measure the solenoid’s B-field at a fixed current \( I_0 \) at various points along its axis with a Gaussmeter; integrating this data gives us \( \Gamma \). Next we apply a current \( I \) to the solenoid and measure it with an ammeter. The only thing remaining then is to find a suitable method to measure the extremely small value of \( \Phi \). This issue will be addressed in the next sections.

**Measuring Changes in the Polarization Angle with one Fixed Polarizer**

To measure a change in polarization angle one monitors the change in light intensity with a photo detector after the light has passed through a fixed linear polarizer. (See the Figure 6.2. below.) The photo detector produces a current that is directly proportional to the observed intensity. This current is then passed over a 50 Ohm resistor to produce a voltage. This voltage will be directly proportional to the light intensity, \( I_p \):

\[ V \propto I_p \]  

(6.4.)

Figure 6.2. Linearly polarized light passes through a (linear) polarizer. Since this polarizer has been rotated by an angle \( \phi \) with respect to the beam’s original polarization axis, the light’s intensity will be reduced by \( \cos^2(\phi) \) when it finally reaches the detector.

Malus law states that:

\[ I_p = I_o \cos^2(\phi) \]  

(6.5.)

where \( \phi \) represents the angle between the plane of polarization, i.e., the plane of vibration, in a light beam and the axis of the optical polarizer.

Since the rotation due to the Faraday Effect is usually very small, let’s express \( \phi \) as the sum of two terms where \( \phi_0 \) represents the polarization axis of the incident light beam and \( d\phi \) represents the small rotation due the Faraday Effect:
\[ \varphi = \varphi_0 + d\varphi \]  \hspace{1cm} (6.6.)

(Though \( \varphi_0 \) can be adjusted arbitrarily when setting up the experiment, during the experiment it will remain constant.)

The change in intensity due to the Faraday Effect, \( d\varphi \), then becomes:

\[ I_p = I_o \cos^2(\varphi_0 + d\varphi) \]  \hspace{1cm} (6.7.)

Measuring \( d\varphi \) directly, using the method shown in Figure 6.2, requires a careful monitoring of \( I_o \) since \( I_p \) directly depends on it. In addition, the "sensitivity" of \( I_p \) depends on \( \varphi_0 \). For example, for \( \varphi_0 = 0 \) or \( \varphi_0 = \pi/2 \), \( I_p \) is not very sensitive with respect to \( d\varphi \). (Proof left to the reader.) The best sensitivity can be obtained by setting \( \varphi_0 = \pi/4 \). In this case:

\[ I_p \approx \frac{I_o}{2} \left( 1 - d\varphi \right) \text{ for small } d\varphi \]  \hspace{1cm} (6.8.)

**Measuring Small Changes in the Polarization Angle with an Optical Bridge**

An improvement over the first setup is shown in Figure 6.3. The polarizer now has been set at 45 degree with respect to the incident beam and its polarization. At this setting, the beam will be split into two beams: ½ of the intensity will be transmitted and (the other) half reflected. Hence, the 2 detectors, \( D_A \) and \( D_B \) should measure identical intensities.

![Figure 6.3. A linearly polarized beam, incident from the right, intercepts a Glan Taylor Calcite Polarizer, shown in the middle of the picture. This polarizer passes the p-polarization component and reflects the s-polarization component at a 68° angle to the incident beam. If the incident beam is polarized at exactly 45° with the p-polarization component, then the intensities observed at the detectors \( D_A \) and \( D_B \) will be identical. Changes in the polarization angle of the incident beam result in an increase of intensity in one detector while producing an equivalent decrease in the other.](image)
If we now again introduce a small change in polarization, \( d\phi \), (due to the Faraday Effect) we can see that the intensity at the one detector will increase by roughly the same amount as it will decrease by the other.

\[
I_{PA} = \frac{I_o}{2} (1 - 2d\phi) \\
I_{PB} = \frac{I_o}{2} (1 + 2d\phi)
\]  
(6.9a.)

(6.9b.)

Therefore, if we monitor the difference (voltage) between \( D_A \) and \( D_B \), i.e., we get:

\[
\Delta V \equiv V_A - V_B \propto 2I_o d\phi
\]  
(6.10.)

Though we have gained a factor of 4 in "sensitivity," to convert our voltage readings back into an angle, we still need to know (among other things) the exact value of \( I_o \) which can be obtained from the sum of the voltages at \( D_A \) and \( D_B \):

\[
\Sigma V \equiv V_A + V_B \propto I_o
\]  
(6.11.)

Finally, the change in polarization angle can be found from the ratio of the difference and sum voltages:

\[
d\phi \cong \frac{1}{2} \frac{\Delta V}{\Sigma V} = \frac{1}{2} \frac{V_A - V_B}{V_A + V_B}, \text{ for small } d\phi
\]  
(6.12.)

Not only is this equation (finally) independent of \( I_o \), from this equation, the change in polarization angle (in radians) can be directly calculated from the voltages at the two detectors without any further calibrations!

How does \( \Phi \) relate to \( d\phi \)? Since \( \Phi \) is the rotation angle due to an applied B-field, it can be determined by measuring \( d\phi \) twice, once with an applied B-field and once with the B-field being zero:

\[
\Phi \equiv d\phi \bigg|_{B=0} - d\phi \bigg|_{B=0}
\]  
(6.13.)

**Measuring Small Voltage Changes Using a Lock-In Amplifier**

To measure the small voltage changes, i.e., \( \Delta V \), we will use a computer interfaced Stanford SR810 Lock-in Amplifier, abbreviated as “LIA.”

To understand how a LIA works, consider the following brief description of a “generic” experiment:
First, the signal of interest, in the above example the intensity of the laser light, is “chopped” or modulated at some fixed reference frequency, \( f_{\text{ref}} \), before it is sent to the actual experiment. This reference frequency is provided either by the function generator section of the LIA; or, as is sometimes the case, it can also come from an external function generator, in which case the LIA phase-lock-loop circuit (PLL) is used to “lock-in” on the external frequency and to regenerate it. The only requirement for the reference frequency is that it remains “stable” with time during the data acquisition cycle.

Second, after the intensity of the modulated signal has “somehow” been affected by the experiment, its detected voltage, \( V_{\text{det}} \) (or current) signal is sent back into the LIA signal input section where it is amplified.

Finally, the LIA mixes the reference signal with the detected signal in a process called “heterodyning.” The result of this process is that only signals with frequency components that are “close” to the reference frequency are passed and all others are rejected. In other words, heterodyning acts like a narrow band pass filter passing only signals near the reference frequency. Next the signal is integrated, i.e., averaged, over multiple modulation cycles to further reduce noise. In terms of the band pass filter analogy, the duration of the averaging (which is adjusted through the LIA’s time constant settings,) is inversely related to the width of the band pass filter, i.e., the longer we average, the narrower the band pass filter becomes.

Ultimately, what the LIA measures is the RMS voltage (or current) of the AC signal at the reference frequency. As has been explained, it accomplishes this by filtering, averaging and amplifying the signal. The LIA’s function (and functionality) is “somewhat” similar to a typical radio receiver which is able to receive a modulated signal buried amid lots of noise using a similar heterodyning technique. To learn more about heterodyning see H&H pages 885 to 897; a good introduction to LIAs can be found at the LIA manufacturer’s website: [http://www.thinksrs.com/downloads/PDFs/ApplicationNotes/AboutLIAs.pdf](http://www.thinksrs.com/downloads/PDFs/ApplicationNotes/AboutLIAs.pdf)

After the brief explanation of how the LIA works, consider why and when to use a LIA. LIAs are typically used to accurately measure very small signals that change little with time, i.e., signals in the Hertz range. For example, in our experiment, the signal, i.e., the polarization angle remains constant as long as the applied B-field is not changed. For all practical purposes, during this time interval the detected signal also shouldn’t change and could be considered a “DC signal.” In reality though, temperature and mechanical effects and intrinsic 1/f noise, cause it to drift slowly. Therefore, using a straightforward DC measurement technique makes measuring small changes in a small DC signal difficult and repeating these measurements will only increase the error or uncertainty as the measured values keep on drifting.
Figure 6.5. The diagram on the left shows the original low frequency signal in the frequency domain buried in 1/f noise. The diagram on the right shows the same signal after it has been modulated and shifted to $f_{ref}$.

An alternate way to think of drift and 1/f noise is to look at the experiment in the frequency domain. (See the Figures above.) If a low frequency or DC signal is directly measured with a DC measurement technique it will always be embedded in 1/f noise. On the other hand, if the low frequency signal is modulated to some (higher) reference frequency, $f_{ref}$, it will be shifted in frequency space away from where 1/f noise can affect it to a new frequency where it can be effectively filtered by the heterodyning technique explained earlier. You should now understand why the (optical) DC signal is converted into AC by chopping the laser light.

Finally, a couple closing comments regarding LIAs: Note that so far we have not specified a value for $f_{ref}$ except that it should be much larger than the rate at which we expect the signal to change. Generally, the actual frequency of $f_{ref}$ is not very important as long as it is not near other well known noise frequencies such as 60 Hz, 120 Hz and its harmonics. On the other hand, very large values for $f_{ref}$ may result in the modulator or chopper not being able to keep up and results in producing modulation harmonics and phase lag. In this experiment we found frequencies between 4500 to 5500 Hz to work well.

Another variable that directly affects the noise rejection is the LIA’s time constant’s setting. As already explained, increasing it essentially narrows the width of the LIA band pass filter. With its range from 10 usec to 30000 seconds, it corresponds to the band pass filter’s -3dB point being either a few kHz or a few uHz away from $f_{ref}$. From a practical point of view, to make sense of a reading, generally it should be integrated over about 10 time constants. Therefore, setting its maximum value would require us to wait about 4 days to obtain a single (sensible) reading! As usual a mixture of common sense and trial and error has shown that a time constant of 0.03sec is most suitable for this experiment.

Last but not least, let’s consider a common source of confusion regarding the laser light’s characteristic (optical) frequency: The applied Diode laser light has a wavelength of 635 nm which implies that its intensity oscillates at a frequency of $5.4 \times 10^{14}$ Hz. So far we have not considered the effect of these high frequency intensity modulations on our experiment and simply ignored them. We can get away with that because from an experimental viewpoint, these extremely high frequency modulations can not be observed and appear to the observer as a (constant) DC signal. The laser’s characteristic frequency exceeds the bandwidth at which present day electronics circuits work by a large margin. (Typical signal processing extends only into the Gigahertz ($10^9$ Hz) range.) For this reason the detecting circuit will act like a low pass filter and it outputs a constant DC signal even though the actual laser light is oscillating at a very high frequency.
6.3. Experimental Setup

Circularly polarized laser light at a wavelength of 635nm is emitted from a diode laser. Its intensity is modulated at a reference frequency, \( f_{\text{ref}} \), produced by the SR810 Lock-In Amplifier, between 4500 Hz to 5500 Hz. (Note: the TTL Output is located at the back of the LIA.) The light then passes through the first polarizer \( P_0 \). This polarizer has been aligned so that the resulting linear polarization is at angle of 45° with the second polarizer, \( P_1 \). Furthermore, the first polarizer has been mounted on a precision rotation mount which allows for accurate nulling of the optical bridge composed of detectors \( D_a \) and \( D_b \). The laser light then passes through the sample which in our case consists of distilled water inside a quartz glass tube. The sample has been placed inside of a homebuilt solenoid. When current from the Agilent E3631A power supply flows through the solenoid, the resulting B-field will change the laser light’s polarization axis as it passes through the sample. Finally, the light reaches the second polarizer, \( P_1 \), which also acts as a beam splitter, passing the p-polarization while reflecting the s-polarization component at a 68° angle to the incident beam. The intensity of the p-polarization component is detected by detector \( D_a \) while the...
intensity of the s-polarization component is measured by detector D_B. Both detectors are Silicon detectors, producing an electrical current directly proportional to the applied light intensity. By passing the current through a 50 Ohm terminator a corresponding voltage signal is produced. The difference RMS voltage between the two detectors, ΔV, which is proportional to the change in angle of polarization angle, is measured by the A and B inputs of LIA in its differential, i.e., A-B input, mode. The RMS voltage of the detector D_A is measured with an HP 34401A DVM in its AC mode. (We could (theoretically) read both ΔV and V_A with a single LIA but this approach would require adjusting a large number of settings on the LIA as we toggle back and forth for each measurement which is clearly too cumbersome.)

Note that the LIA, the DVM and the (current) Power Supply can be fully controlled through the computer using the GPIB interface. This allows us to control the solenoid current and then read back all the relevant voltages to calculate Verdet’s constant for water.

6.4. Experimental Procedure and Data Analysis

Overview

To measure the Verdet constant of water 4 different measurements / calibrations must be performed:
1) The B-Field must be measured as a function of position and current.
2) The polarization angle, Φ, as a function of applied solenoid current is measured for the water cell.
3) The polarization angle, Φ, as a function of applied solenoid current is measured for an empty quartz glass cell. This data set allows correcting for the “window effect” at the end of the glass cell.
4) A correction factor, k, is measured to adjust the measured value of Φ to the “real” one.

Each step is described in more detail in the following sections.

B-Field Calibration: Γ

Verdet’s constant depends directly on the strength of the B-field. (See equation 6.3.c.) Therefore, it is crucial that we accurately determine Γ. The measurement of B_x(I_o, x) is straightforward: a longitudinal Gauss probe is inserted at the exact center of the solenoid and the B-field is measured with a gaussmeter along its axis while a fixed current is applied to the solenoid. Unfortunately, this procedure is time consuming. Also, if not done carefully, it can upset the optical alignment of the various components.

For these reasons, you will be given the values from an earlier measurement. You can find them in the spreadsheet called Coil4BField.xls in the U:\pub\statistics folder. The field along the x-axis of the solenoid was measured with a F.W. Bell Model 9200 Gauss meter while a fixed solenoid current of I_o = 1.000 ± 0.001 Amp was applied. The rated accuracy of the Gauss meter is 0.1 Gauss. The x-values correspond to the position along the axis of the solenoid with x = 0 being the middle. The values are accurate to 0.05 cm.

One method to determine Γ from the given data is to fit it to a second order polynomial, i.e., B(x) = a + bx + cx^2. Use the LSQFit2D.xls spreadsheet which can also be found in the U:\pub\statistics folder. Once the coefficients a, b and c have been obtained, you can then obtain the integral of B(x) by (mathematically) integrating the polynomial, i.e., calculate \( \int (a + bx + cx^2) \, dx \).

This approach allows us to accurately apply the limits of integration, i.e. L/2, the length that the water sample in quartz glass cell #4 is exposed to the B-field. It is: L = 9.4674±0.0024 cm.
A warning regarding the $2^{nd}$ order LSQ fit: the entire data set provided clearly does not follow a $2^{nd}$ order polynomial. (Plot the data to confirm that.) Nevertheless, over the area of interest, namely over the length of the sample, it can be modeled using a $2^{nd}$ order polynomial. (How do we know that? Show in your report.)

Having determined $\Gamma$, what is $\sigma_\Gamma$? Performing an error propagation on the integral of the second order polynomial with respect to $x$, $a$, $b$ and $c$ will certainly yield the result. Nevertheless, careful inspection (or thinking) should convince you that a single term in the polynomial is responsible for the majority of the error. Which term is it and why?

In your write-up, state $\Gamma$ and $\sigma_\Gamma$.

**Measuring $\Phi$ vs. the Applied Solenoid Current for the Water Cell and the Empty Cell**

Though we observe $\Phi$ as a function of the applied B-field, we actually measure $\Phi(I)$. We will use the computer’s GPIB interface to apply a current $I$ to the solenoid and then, again through computer’s GPIB interface, we will read back the corresponding voltages at detectors $D_a$ and $D_b$ which will allow us to calculate the changes in polarization angle.

Specifically, the LabWindows program consists of the following components:

1) Load the appropriate instrument driver for each instrument to your project.
2) “Open” i.e., initialize the device driver for each instrument.
3) Configure each instrument.
4) Turn the current in the solenoid off and measure and calculate $d\varphi\big|_{B=0}$.
5) Turn the current in the solenoid on and measure and calculate $d\varphi\big|_{B\neq 0}$.
6) From step 4 and 5, $\Phi(I)$ can be calculated using equation 6.13.
7) Finally steps 3 through 5 are repeated a 100 times to provide $\Phi(I)$ values for $0 < I < 1$ Amp.
8) The drivers for each instrument are closed and the program is terminated.

Below are some more detailed instructions on how to proceed. Nevertheless, before you start programming you should check that the hardware is set up properly.

**Initial Setup Check**

Make sure all the instruments are turned on and remain so. **NEVER TURN OFF THE LASER AS IT REQUIRES AT LEAST ½ HOUR TO STABILIZE!**

If not already in place, put cell #4 with the water inside the the solenoid. (Be careful not to disturb the optical alignments of the components!) Use calipers to make sure it’s indeed in the center!

Check that the HP 3631A current power supply works. See if you can adjust the +25V output current between 0 and 1 Amp. (Since it is being used as a current source, set its output voltage to its maximum value and then adjust the current.)

Next check if the LIA X-display shows a voltage change when you change the current. For a change of about 0.1 A you should see roughly a 20 uV change. (See Table 1 below for the LIA manual settings.) If you don’t see any changes check if the input to the LIA is overloaded; if so change its gain setting till the overload light disappears. Also check your DVM; it should read $8 \pm 2$ mVAC.
### Table 1: Manual SR810 Settings

<table>
<thead>
<tr>
<th>Display</th>
<th>X</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time Constant</td>
<td>3 x 10 usec</td>
</tr>
<tr>
<td>Signal Input: Input</td>
<td>A-B</td>
</tr>
<tr>
<td>Signal Input: Couple</td>
<td>AC</td>
</tr>
<tr>
<td>Signal Input: Ground</td>
<td>Float</td>
</tr>
<tr>
<td>Freq.</td>
<td>4500 to 5500 Hz</td>
</tr>
<tr>
<td>Source</td>
<td>Internal</td>
</tr>
<tr>
<td>Sensitivity</td>
<td>5 x 100 uV (or higher if OVLD is lit)</td>
</tr>
</tbody>
</table>

Now null the optical bridge: See figure 6.7. and carefully turn the “Fine Adjustment” knob on the rotational stage. This rotates polarizer P₀, the one closest to the laser. Turn this knob until the LIA displays a value less than +/- 10 uV. IF YOU ARE UNABLE TO DO SO, DO NOT ADJUST ANYTHING ELSE; SEE YOUR TA IMMEDIATELY!

If you have successfully nulled the optical bridge, proceed to the programming part.

**Step 1: GPIB Drivers**

GPIB used to be almost the only standard for controlling scientific instruments. Though it is being rapidly replaced with USB, it still can be found in most new scientific instruments. Though GPIB was intended to be a standard communications protocol between devices, except for the hardware (note the heavy gray cables that can be “daisy chained” to interconnect various instruments) and low the level interfacing, many of the interface commands are not standard. Therefore, a library of device drivers was created for most scientific instruments making interfacing GPIB devices easier. Unless you want to write your own low level interface which can be a daunting task, you first have to add the device drivers to your LabWindows program.

Start a new project, ideally in a new workspace and new folder. Create a very simple GUI with a Quit command button and generate the basic code for it.

Before you can add the instrument drivers copy them from the U:\pub\LW\InstDrivers directory. Specifically, copy the following three folders into your current LabWindows project folder:

- U:\pub\LW\InstDrivers\HP34401A (Driver for DVM)
- U:\pub\LW\InstDrivers\HP3631A (Driver for Powersupply)
- U:\pub\LW\InstDrivers\SR830 (Driver for SR810 LIA)

Note: the device driver for the SR830 and will also work with the SR810 and so we will use it instead.

Now add the instrument drivers to your current project. On the upper left panel right-click on the project name and select ‘Add Files to Project.’ Select for the “File Type:” “Instrument (*.fp) files and then select the three files in the above copied folders and add them to your project:

- hp34401a.fp
- hp3631a.fp
- sr830.fp

You now should see the three instruments listed in the bottom left panel beneath the library functions.
Steps 2 and 8: Initialize Each Instrument / Close Each Instrument

Before you can communicate with an instrument you must “open” and initialize the device driver. Each instrument must have a unique instrument address assigned to it. Though this number is arbitrary and can be changed, to make the programming easier we have clearly labeled each instrument with its unique address. Please do not change it!

<table>
<thead>
<tr>
<th>Instrument Type</th>
<th>Manufacturer</th>
<th>Product Number</th>
<th>GPIB Address</th>
</tr>
</thead>
<tbody>
<tr>
<td>DVM</td>
<td>HP / Agilent</td>
<td>HP34401A</td>
<td>3</td>
</tr>
<tr>
<td>Power Supply</td>
<td>HP / Agilent</td>
<td>HP3631A</td>
<td>7</td>
</tr>
<tr>
<td>Lock-In Amplifier</td>
<td>Stanford Research</td>
<td>SR810</td>
<td>11</td>
</tr>
</tbody>
</table>

Since each instrument will be used during the entire time the program runs, the best place to initialize them is right in “main” before the “DisplayPanel” function, as shown below:

```c
hpe363xa_init ("GPIB::7", VI_ON, VI_ON, &insthandle_current);
hp34401a_init ("GPIB::03", VI_ON, VI_ON, &ihandleDVM);
sr830_InitNReset (1, 11);
DisplayPanel (panelHandle);
RunUserInterface ();
```

Instead of copying the above code, click on the library functions of the instruments in the bottom left panel and insert them this way. Make sure the GPIB addresses agree with the table above. Use LabWindows to declare the two instrument handler variables as they are of “ViSession” type. They should be global variables!

Finally, the drivers must be shut down properly when your program terminates. This is accomplished through the individual instrument drivers “close” functions. Not closing an instrument properly (as happens during a system crash) may prevent it from working the next instance. If that is the case, turn the instrument in question off and then on again.

The instrument close functions can be inserted into the call back function that closes your program, right before the “QuitUserInterface” function as shown below:

```c
hpe363xa_close (insthandle_current);
hp34401a_close (ihandleDVM);
sr830_close ();
QuitUserInterface (0);
```

You should execute your program at this point to make sure the instruments are connected and that your code is working.

Step 3: Configuring the Instruments

Before you can read data back from the instruments each setting must be configured. Add a command button to your GUI and generate its callback function that will control the instrument configuration and data acquisition cycle.

In this callback function configure the instruments as follows:

**HP 34401A Multimeter:** (See under Configuration Functions:)

<table>
<thead>
<tr>
<th>Function</th>
<th>Settings</th>
</tr>
</thead>
<tbody>
<tr>
<td>conf</td>
<td>Function: Voltage AC</td>
</tr>
<tr>
<td></td>
<td>Auto Range: On</td>
</tr>
</tbody>
</table>
Resolution: 6 ½ Digits

**confTrig**
- Trigger Source: Internal
- Leave rest default

<table>
<thead>
<tr>
<th><strong>HP HP363xA DC Power Supply</strong></th>
<th>(See under Configuration Functions \ Config HP3631A:)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Function</strong></td>
<td><strong>Settings</strong></td>
</tr>
</tbody>
</table>
| configOutput3631 | Outputs: Enable
- Tracking: Disable |

Note: make sure these functions above refer to the appropriate instrument handle declared in the previous section!

<table>
<thead>
<tr>
<th><strong>SR810 Lock-In Amplifier V1</strong></th>
<th>(See under Configure Functions)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Function</strong></td>
<td><strong>Settings</strong></td>
</tr>
</tbody>
</table>
| SetFrequency | Set (Output) Frequency: Some value between 4500 and 5500 Hz
- Reference Frequency: Internal |
| GainNTime | Gain: 500 uV
- Time Constant: 30 msec |
| InputCoupling | Input Configuration: Differential Input A-B
- Input Coupling: AC
- Input Shield Grounding: Float |
| SetFilterSlope | Low Pass Filter Slope: 12 dB / oct |
| AutoSetup | Auto Reserve: On
(Leave all the others Off.) |

**Steps 4 and 5: Acquire Data**

Use a for-loop to increment the current through the solenoid from 0 to 1 Amp in 0.01 Amp increment steps and acquire the data:

1. Set the current of the power supply to 0 Amps using the configCurrVolt3631 function; set the voltage of the +25V output terminal to its maximum, 25V value.
2. Delay the voltage readings by ½ second by calling the LabWindows “Delay” function with an argument of 0.5.
3. Read $V_A$ using the HP34401A Multimeter / Data Functions / SingleMeas function; the value returned by “Measurement” is the voltage read.
4. Finally, read $\Delta V$ from the SR810 using the SR830 Lock-In Amplifier / Measure / ReadInputs and select Input Channel X; the value returned by “Value” corresponds to $\Delta V$.
5. From $V_A$ and $\Delta V$ you should be able to calculate $V_B$ and, hence, $d\phi \big|_{B=0}$

Immediately following these commands, repeat them once more with the exception that this time, in step 1, the current is set to the value specified by the loop. These second set of 5 commands will give you the data to calculate $d\phi \big|_{B \neq 0}$. From $d\phi \big|_{B=0}$ and $d\phi \big|_{B \neq 0}$ you should be able to calculate $\Phi$, see equation 6.13.

**Data Files**

Analyzing the acquired data directly in LabWindows can be a difficult task. Furthermore, it will tie up the equipment. Instead, we want you to write all your acquired data to a comma-separated (CSV) data file to be analyzed in Excel. (For a more detailed description on how to write to an Excel file see C & LabWindows Fundamentals, sections 11.2 through 11.5, specifically page 63.)
Writing data to a file that can then be read into Excel involves the following three steps:

1) Create a file pointer and open the file.
2) Write data to the file.
3) Close the file.

Below are the details.

Before you can write data to a file you must declare a FILE pointer variable. It is called *fptr in the example below. You probably want to declare this as a global variable, i.e., outside of main.

```c
static FILE *fptr;
```

In main open the file for writing using the fopen function.

```c
fptr = fopen("test1.csv", "w");
```

The first argument specifies the file name. The file will be created in the current LabWindows project directory. Be sure to keep the CSV file extension if you want to open it in Excel when you double click on it. Note: if a file with the same name already exists, it will be overwritten. Therefore, either copy the existing data file to a different file or directory or change the file name in the line above!

Every time the following lines are executed, one row of data is appended to the file:

```c
fprintf(fptr, "%E,%E,%E,%E,%E,%E,%E,%E\n",
    i, va0, val, dv0, dva, dphi0, dphi1, dphi1 - dphi0);
```

Each %E is directly "linked" to a related variable that has been listed following the closing quotation mark. In this particular example, the first %E specifies to write the variable i (in scientific notation) to the data file, followed by a comma. (The comma forces the next value to be written into the next column.) The subsequent %E specifies to write the variable val0 followed by a comma to the data file and so on. The very last %E relates to the calculated value of dphi1 – dphi0 and is followed by \n, signaling then of the line, i.e., row of data.

Close the file before terminating the program.

```c
fclose(fptr);
```

You may do so right before the QuitUserInterface statement.

**Data Acquisition / Analysis**

Run the data acquisition twice; first with the water cell and then with an empty cell. While you are at it, also determine $\sigma_\Phi$ for the two cases by reading $\Phi$ 100 times for the same (arbitrary) current. (Modify your program: instead of setting the current to the increasing current value of the for-loop, set it to some arbitrarily chosen value between 0 and 1 Amp.)

When you remove the water cell and insert the empty cell, you might have to null the optical bridge by rotating the fine adjustment micrometer on polarizer P0. (See Figure 6.7.) Use the calipers to verify that the cells are located at the very center of the solenoid.

Plot the two data sets of $\Phi$ vs. I. A quick visual inspections should show a linear relations ship; if not check your code or ask your TA for help.

Analyze your data with the Excel least squares fit spreadsheet LSQFit.xls from U:\pub\statistics folder. Fit $\Phi$ vs. I. Use $\Phi$ as the dependent variable and I as the independent one. (Why?) Determine the slope, i.e., find $d\Phi/dI$, for both cells. (Remember, if you used equation 12 to
calculate Φ, then your answer will be in radians / Amp!) Use the σΦ obtained previously and see how well it satisfies the criterion that the reduced χ² should be close to unity. If this is not the case, can you think of a reason? In your report clearly state dΦ/dI for both cases, its uncertainty, σΦ used for the fit, the reduced χ² and the number of data points in the fit.

Since numbers by themselves do not convey a meaning, use the following example to gain a better understanding to what amazing accuracy this experiment is able to measure a change in angle: for your σΦ, what is the smallest distance (of arc), σs, that it is able to measure at a radius of 1000 m? (Use σs = r σΦ.)

The quartz glass windows of the water cell also provide a medium for the Faraday rotation. Their not-insignificant contribution must be subtracted to measure the rotation effect from the water only.

If we have two media, such as quartz glass and water, equation 6.3.c. becomes:

\[ \Phi_{Glass\&Water} = v_{Glass} \Gamma_{Glass} I + v_{Water} \Gamma_{Water} I \]  \( \text{(6.16.)} \)

For the empty cell, we get:

\[ \Phi_{Empty} = v_{Glass} \Gamma_{Glass} I + v_{Air} \Gamma_{Air} I \]  \( \text{(6.17.)} \)

If we neglect the Faraday Effect on the air in the empty cell, it is 1000 times smaller than that of water or glass, then we see that:

\[ \frac{d\Phi_{Water}}{dI} = \frac{d\Phi_{Glass\&Water}}{dI} - \frac{d\Phi_{Empty}}{dI} \]  \( \text{(6.18.)} \)

In other words, from the slope of the two data sets, and equation 6.3.c. and the previously obtained values for Γ, a "pretty good" value for the Verdet constant for Water (and its error) can be obtained. (It still can be improved though – see the next section.) Check your answer; it should be roughly 0.01 minutes of arc / Gauss cm. State your value and its uncertainties. What was the % contribution of the quartz glass windows on your final result?

**Correction Factor, k, for dϕ**

Equation 6.12. relates dϕ to the measured difference and sum detector voltages. It is based on the assumption that the Glan Taylor Calcite polarizer and beam splitter, P₁, completely separates the p and s polarization components for the detectors D_A and D_B. Our own data and published results reveal limitations. Therefore, equation 6.12. needs to be augmented with a correction (fudge factor,) k:

\[ d\varphi_{real} = kd\varphi_{measured} = \frac{1}{2} k \frac{\Delta V}{\Sigma V} = \frac{1}{2} k \frac{V_A - V_B}{V_A + V_B} \quad (\text{for small } d\varphi) \]  \( \text{(6.19.)} \)

This correction factor can be determined by rotating the first polarizer, P₀, by a known amount, dϕ₀lab and then comparing it to the value obtained from equation 6.19, dϕmeasured. Our previous data collection over the applied B-field spanned a polarization angle change of \( \frac{\pi}{4} \). We will now use the fine adjustment on the high precision rotation mount to rotate P₀ by the same amount while observing dϕmeasured. (See figure below.)
Before you take the correction factor data, remove the empty cell and reinsert the water cell in the center of the solenoid. Null the optical bridge with the rotation mount’s fine adjustment. Turn the solenoid current off for these measurements.

One complete 360° turn of the fine adjustment knob corresponds to a polarization angle rotation, \( d\phi_{\text{real}} \) of exactly 1°. Furthermore, this knob has been marked with 25 equally spaced lines, each indicating another polarization angle change of 1/25°. Therefore, the required ¼° calibration range corresponds to a rotation over only 6 to 7 of these markings.

Write down the values for \( \Delta V \) and \( V_A \) (as observed on the LIA and DVM) and \( d\phi_{\text{real}} \) as you carefully turn the fine adjustment micrometer. Calculate \( d\phi_{\text{measured}} \) and fit it to \( d\phi_{\text{real}} \) and find \( k \) and its uncertainties. (Remind yourself that the units for \( d\phi_{\text{measured}} \) are in radians.) What should \( k \) be for an ideal polarizer / beam splitter? Under no circumstances should you off more than 20% from this ideal value.

The previously obtained value for the Verdet constant of water was based on determining \( d\Phi/dl \). Prove that the correction factor \( k \) can directly be applied to the previously obtained results and calculate your final result. (Of course, you need to account for the new uncertainties.) State your final value for the Verdet constant of water and its uncertainty. Compare it to the most accurate value found in literature of 0.0115 min / G cm at 20° C and at a wavelength of 632.8 nm.

6.5. Write-Up

In addition to describing your experiment, analysis and results be sure to answer all questions raised in this text. In addition, discuss how the quartz glass window and \( k \)-correction factor affected your final result and its uncertainty.

6.6. Acknowledgments

I like to thank Professor Crowell for his suggestions and insights. Two former MXP students, Andrew Stewart and Kienan Trandem built some of the equipment; they tested the experiment during its development stages and have drawn some of the diagrams used in this write-up. Finally, the Physics Machine shop, specifically John Kilgore and Roger Olson advised us and machined some of the equipment. The original idea for this paper came from a paper from Don Heiman at
Northeastern University. Additional papers reviewing and comparing various methods for determining the Verdet constant are listed in the reference section.

7 J-Y Fan et al. (2003), "A study on transmitted intensity of disturbance for air-spaced Glan-type polarizing prisms". Optics Communications 223 (1-3): 11–16.