Photon Counting Statistics and Coherence Time Experiments Using a He-Ne Laser

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Introduction

Our experiments both have much to do with the coherence of laser light. The first experiment, which is now finished, examined the counting distributions produced by photons from our He-Ne laser over “long” and “short” timescales, long and short relative to the coherence time ($\tau_c$) that is. It should be noted here that our short timescale measurements were limited by equipment speed restraints, and we were forced to simulate the short timescale counting scenario. I will however present an argument for why this simulation should, and did, yield the same statistical counting distribution as would have been attained if we would have had access to equipment fast enough to count photons on timescales much smaller than a nanosecond. A skeptic might point out that here our experiment was really to test the validity of this argument and the simulation against the predetermined short timescale counting distribution which has been known for many years.

The second experiment (now under construction) is an interferometer, which we will use to directly measure the coherence time of our laser.

Coherence

The definition of a coherent beam is as follows: any two points on a coherent electromagnetic wave separated by a fixed distance, have a time constant phase difference. The beam length, for which the phase difference remains constant is called the coherence length $L_c$, and the time that it takes for a hunk of beam exactly that long to pass by a fixed point is the called the coherence time $\tau_c$. This concept may seem strange (it certainly did to me) to someone used to thinking of lasers as monochromatic, coherent sources (using both terms rigidly), but after a formal discussion of coherence and some notes on the limitations of lasers this should make sense. The way to quantify coherence is as follows: Consider a perfectly coherent beam of electromagnetic radiation, and an ideal master clock, the master clock ticks independently of the electromagnetic wave, and the wave keeps in step with the clock such that at each clock tick the wave is in the same position in phase space as it was when the clock ticked the time before, and this perfect synchrony keeps on into infinity, such that at tick number one the wave is in the same phase state (trough, crest, in-between, etc) as it will be at tick number one billion, it’s a perfect imaginary system. Now consider the same ideal master clock ticking alongside a real laser beam. For this case the period of the master clock is the reciprocal of the
average frequency of the laser. The two cases are pretty much equivalent on length scales of a few million wavelengths, but after a few nanoseconds the master clock and the wave are no longer perfectly in phase. The correlation function relating the wave’s phase ideal to the clock is:

\[ C(\tau) = \frac{1}{2T} \phi(t) \ast \phi(t + \tau) \, dt \quad \text{where: } T >> \tau \]

Here \( \tau \) is different from the coherence time \( \tau_c \), all is measured in units of master clock ticks, and \( T \) is not any kind of period, just an amount of time. In the above equation the phase of the wave (with reference to the ideal clock) at some time \( t \) is multiplied by the phase of the wave (again with reference to the ideal clock) at some time \( \tau \) (clock ticks) ahead of \( t \), this product is integrated over some time period very long compared to their offset \( \tau \).

*Probably the best way to visualize the argument of the above integral is to think of two long, identical, imperfect waves, then offset them (fast forward one ahead by some amount of time \( \tau \)), then integrate their phase product (with reference to some master clock) over some long time period 2T.*

If the wave and the clock were perfectly separated in phase everywhere in time then every addition to the integral would be positive, but since the wave is randomly ahead or behind the master clock in phase space (a crucial fact due to the way lasers behave), at \( \tau \) clock ticks ahead (assuming \( \tau > \tau_c \)), positive and negative amounts are added randomly to the sum resulting in an integral that reaches zero. If the correlation function approaches zero then the wave is out of phase with the clock by a significant amount \( \sim \pi \). A flat correlation function would indicate phase correlation, as can be seen
by setting $\tau = 0$, the result is an integral of $\phi(t)^2$, and that is a constant, positive value. The formal definition of coherence time then is the characteristic decay time of the correlation function, the time it takes for the correlation function to decrease by a factor of e. All this happens because the laser is not perfectly monochromatic; a long wave from a laser squirms back and forth a bit in frequency space. We actually watched our He-Ne do this under a spectrum analyzer. The coherence time of a laser (or any other source) is thereby related to the effective frequency bandwidth $\Delta \nu$ by the equation:

$$\tau_c = 1/\Delta \nu$$

But Lasers have only one frequency; a red He-Ne is never green! The frequency spread for a laser is small, but finite, if it were zero, the coherence time would be infinite. The atoms in a laser plasma are moving randomly about due to thermal excitation, and when each emits a photon, Doppler shifts come into play resulting in a myriad of different frequencies all in a narrow range, in a laser cavity usually a handful of the many frequencies (these being the small percentage directed along the cavity length) find that the length of the cavity is just the right length for them to set up standing waves along the cavity (and then go by the fancy code name: ‘longitudinal cavity modes’), become amplified, and then get emitted through the laser aperture. Our He-Ne under a spectrum analyzer proved to settle into only two cavity modes after it was warmed up, the frequency difference between these modes ($\Delta \nu$) was measured to be $\sim 10^{14}$Hz, corresponding to a wavelength range ($\Delta \lambda$) of .02nm, and a coherence time of about one and a half nanoseconds. As our laser warmed up, the cavity expanded and the spectrum “walked” through several different groups of modes until settling into two. Lasers get very close to their theoretical limits when a single cavity mode can be isolated with a frequency bandwidth of $\sim 20$Hz, as in special He-Ne lasers that have coherence times of .05 seconds, that’s a coherence length of over nine thousand miles!

**Counting Statistics and Sampling Times**

Using the definition given above, we can say with certainty that the phase difference between any two points on a beam is constant over a distance interval smaller than the coherence length of the beam. However, during this time the intensity of the beam tends to fluctuate randomly. This is a result of Heisenberg’s uncertainty principle, which states that we cannot know the intensity and the phase of a beam with complete accuracy at the same moment. Hence for times less than the coherence time, the beam’s phase is constant but its intensity is not.

The following derivation of the Bose-Einstein distribution we’ve pieced together from references 3 and 4 with much help from Dr. Crowell our advisor.

For a laser (on time scales longer than the coherence time) the intensity is constant. More specifically, over these “long” time periods the random fluctuations in intensity can cancel each other out. A good analogy for a laser with a constant photon emission rate is a radioisotope with a constant radioactive decay rate. Both cases obey a Poisson distribution: The probability $P(n)$ of observing $n$ events, where $<n>$ is the average, is given by:
\[ P(n) = \frac{<n>^n e^{-<n>}}{n!} \]  

(1)

The intensity is thereby given as:

\[ I = \frac{<n>h\nu}{tA} \]  

(2)

where \(<n>/t\) is the average number of photons per unit time, \(h\nu\) is the energy of each photon, and \(A\) is the area of the detector. We now want to put intensity in terms of number of photons, in solving for \(<n>\), we get \(<n> = \alpha I\) where we let \(\alpha = \frac{tA}{h\nu}\). Substituting for \(<n>\) in equation (1) yields:

\[ P(n) = \frac{(\alpha I)^n e^{-\alpha I}}{n!} \]  

(3)

This equation gives the predicted distribution for time scales longer than the coherence time in this experiment.

Next, we can generalize the above equation for light sources of varying intensities. For a light source whose intensity obeys a given probability density function \(P(I)\), equation (3) can be multiplied by \(P(I)\) and integrated over all intensities, giving:

\[ P(n) = \int \left( \frac{(\alpha I)^n}{n!} \right) e^{-\alpha I} P(I) dI \]  

(4)

This equation is known as Mandel’s formula.\(^3\)

On time scales less than the coherence time of the beam, the photons are distributed quite differently. Due to an effect known as “photon bunching,” the probability of detecting a photon doubles immediately after one has been detected.\(^2\) This effect was experimentally verified by Hanbury, Brown, and Twiss in 1956, using a setup similar to our own proposed setup for our second experiment (the interferometer we’ll use to measure coherence length). In their famous experiment a beam of photons was split into two parts using a partially transparent mirror (beam splitter). Two separate detectors at equal lengths from the mirror then detected the two beams. By comparing the results from both detectors, they found that the intensity fluctuations in the two beams were correlated.\(^4\) This of course meant that the photons were arriving at the beam splitter in groups. In other words, the experiment proved that the presence of one photon increases the likelihood that another will be present in a bunch. The shape of the Bose-Einstein distribution, figure 2, evidences this idea. It can be seen from the plot that the most likely bunch-size to observe is zero photons, the next most likely observation will be one photon, and then two, three, and so on, suggesting that the bigger the bunch, the less likely is to occur.
Now for the B-E distribution itself, in considering photon probability densities it is known that all photons must obey the Boltzman energy distribution:

\[ P(E) = C_1e^{-C_2E} \]  

(5)

where \( E \) is the energy of the photon and \( C_1 \) and \( C_2 \) are constants. Since energy is proportional to the square of the electric field, which in turn is proportional to the intensity of the beam, we can replace \( E \) in equation (5) by simply by choosing new values for the constants \( C_1 \) and \( C_2 \). To normalize this probability density function, we must note that:

\[ \int C_1e^{-C_2I}dI = 1 \]  

(6)

Evaluating this integral gives \( C_1=C_2 \). Furthermore, the average intensity \( <I> \) is defined according to the equation

\[ \int C_{1}e^{-C_1I}dI = <I> \]  

(7)
Using integration by parts, this integral gives C as the reciprocal of the average intensity. Combining these two results:

\[ P(I) = e^{-I/<I>/<I>} \]  \hspace{1cm} (8)

This is the probability density function for the intensity of the beam. Substituting this into Mandel’s formula gives:

\[ P(n) = \int \left( (\alpha I)^n/<I>/n! \right) e^{-I/<I>/}<I> dI = (\alpha I)^n/<I>/n! \int I^n e^{-I/(\alpha + 1/<I>/)} dI \]  \hspace{1cm} (9)

Finally, evaluating this integral using an integral table and simplifying gives

\[ P(n) = (<I>/\alpha)^n/(1+<I>/\alpha)^{n+1} \]  \hspace{1cm} (10)

And Voila, the Bose-Einstein distribution for observations made on timescales shorter than the coherence time of an incident beam, this distribution came about because of the fact that we made no assumptions about any special arrangement of the photons in their energy levels in equation 5, we just expressed photon energy in terms of average intensity, allowing any number of photons to be described as having the same energy level, this makes sense, since photons are bosons.

**Experiment**
In observing a laser beam on such short timescales, one would detect lumps of photons at random times, according to the HBT experiment. The size of these lumps would increase along with their rarity. Most of the time no photons would be observed. Our approximation to this scenario used a ground glass disk with one irregular surface. The beam was focused on the far side of the disk (the irregular side) and the emerging beam was randomized into clumps, producing a speckle pattern. A small motor slowly turned the disk, and the speckle pattern swept past the pinhole on the face of the dark box that contained our Photon counting head. The pattern was dim enough such that the space among the speckles (the majority of the pattern area) corresponded to no photon counts when these dark patches fell onto the detector. These dark patches were actually tiny, faint speckles. My theory for why the speckle distribution approximated the B-E distribution is as follows: I’d guess that the distribution of surface irregularity sizes would be pretty normal, probably fitting a Gaussian distribution fairly well. I’d also wager that the resultant speckle pattern has a speckle size which fits a normal distribution as well, both of these are after all products of a sandblasted surface and therefore fairly randomized properties. Imagine our speckle pattern as fitting a normal distribution, there is some average speckle size, and the speckle sizes smaller and larger than the mean are less and less likely to be present as the size extremity under consideration increases.

Fig. 4

The speckles to the left of the mean are very faint, and to the far left of the mean they are almost indistinguishable from darkness. In our attenuated beam, the fainter speckles disappeared, leaving us with just the tail of the right side of the Gaussian distribution
depicted above. Does such a distribution look familiar? It looks like a good approximation to a Bose-Einstein distribution to me.

To sum up; in observing a laser on timescales shorter than the coherence time (short enough that the photon bunches become apparent) what one would see would be...mostly nothing, as zero would be the most frequently observed photon bunch size, but bunches would eventually arrive, at random times, with a bunch size frequency dictated by the B-E distribution, rarely would you see a big bunch (or two medium-sized bunches in rapid succession) mostly you would see small bunches, or nothing at all.

In observing our B-E simulation (on timescales smaller than the speckle sweep time, which is the time it takes for a speckle to sweep over the pinhole allowing the photon counting head to register some hits) one would see (from behind the pinhole)...mostly nothing, as most of our speckle pattern was darkness, but eventually photons would arrive, in bunches, due this time to large timescale speckle effect, and not the statistical behavior of bosons as in the previous ideal example. The coherence time of our simulation is not known. However, we do know a lower bound for the simulated coherence time, namely the speckle sweep time, analogous to the average time between photon bunch arrivals (certainly smaller than the coherence time) this turned out to be about a millisecond, and our sampling times were on the order of 10μsec. Speckles would fall onto the pinhole allowing flashes of light to poke through, this would happen at random times, due to the fact that the surface irregularities of the glass are randomly spaced, yielding a speckle pattern whose speckles are randomly spaced. So as the pinhole “travels” through the pattern it detects speckles at random times. Sometimes one would see big bunches of photons come through the pinhole, but only very rarely, most of the time when a bunch would come through the pinhole it would be small, as the average speckle size is so small due to the beam’s energy diffusion (due to the fact that the glass spreads the beam over a large area making the pattern faint except for the rare, large speckles) that it registers no hits on the counter. It can be seen from the qualitative description of these two scenarios as well as the prior description of the truncated Gaussian distribution resulting from our B-E simulation, why the simulation should work. Presented below is a plot of some of our preliminary results from the simulation.

Fig. 5
Noise reduction (actually noise avoidance) was considered in our experiment, as our Photo Multiplier Tube (PMT) always recorded some dark counts, the dark count rate rose as the tube warmed up, to monitor this behavior, a dark count trial was taken immediately after each set of trials to make sure that the dark count rate was indeed still negligible. Dark count rates for this experiment ranged from ~160 – 400 counts per second, our sampling times were on the order of milliseconds however, so it can be safely assumed that dark counts due to shot noise etc. was never a factor in our results.

More Preliminary Results
Experiment 2

Our setup will consist of a Michelson- Morley type interferometer (without the spinning platform) with one of the mirrors mounted on a piezo electric motor to thrust it back and forth along the axis of the incoming beam a distance greater than the wavelength of the red light. At our detector we should see the intensity fluctuating between the initial intensity of the single beam, and zero. This fluctuation should occur as long as the path length difference between the two arms is shorter than the coherence length. The other mirror (a three way) will be mounted on a shaft turned by a stepper motor to allow us to precisely change its distance to the beam splitter.

Fig. 6

References


