Abstract

The purpose of our experiment is to measure the statistical distribution of photons emitted from a laser on timescales longer and shorter than the laser’s coherence time ($\tau_c$). Since the coherence time of a typical laser is on the order of 1 ns, we cannot measure the distribution on this timescale due to limitations in the speed of our equipment. However, we can simulate the behavior of a laser on this timescale by sending the beam through a spinning ground glass disk, which approximates the situation to a surprising degree. For a basic He-Ne laser, the distributions on timescales longer and shorter than $\tau_c$ should be Poisson and Bose-Einstein, respectively. For an Argon laser, the distributions should be more complex due to the multiple longitudinal and transverse modes of the beam.

The second part of the experiment involves constructing a standard Michelson Interferometer to obtain an estimate for the coherence length of a He-Ne laser. This can be done by measuring the point at which the laser fails to interfere with itself at the detector. This path length difference is, by definition, the coherence length of the laser.

Introduction

The first visible light laser was developed by Theodore Harold Maiman and was announced to the world on July 7, 1960\textsuperscript{4}. This laser used ruby (a combination of Al$_2$O$_3$ and Cr$_2$O$_3$) as its medium, although the He-Ne laser followed soon after\textsuperscript{4}. Since then,
Lasers have been developed and refined at a staggering pace, and have been applied to many different elements, including all of the noble gases\(^4\).

Due to their high levels of coherence and power, lasers have extended beyond the realm of science and into our everyday lives. Though they are perhaps best known by the public for their ability to read CDs, they also have a huge number of applications in the fields of industry (especially microchip technology), medicine, and national defense just to name a few.

**Theory**

The lasing phenomenon is made possible by a process called the stimulated emission of radiation. This occurs when a photon interacts with an excited atom, causing it to drop down to a lower energy level. The photon given off by the atom then has the same phase, direction, and polarization as the initial photon\(^4\). By repeating this process many times, one can produce a highly coherent beam of light.

Inside a laser, the stimulated emission occurs in a resonant cavity with mirrors at both ends (one of which is partially transparent). The light frequencies that form standing waves in the cavity are then amplified by stimulated emission and emitted from the laser\(^4\). For standing waves that differ by \(\lambda/2\) within the cavity, the frequency difference is given by \(\Delta v = c/2L\) where \(L\) is the length of the cavity\(^4\). These standing waves form the longitudinal “modes” of the laser. Since the bandwidth of the atomic transitions is wider than the resonant modes, there may be several such modes formed in the laser’s cavity. Ideally, the total width of the modes (\(\Delta v\)) then determines the coherence length \(l_c\) of the laser according to the equation \(l_c = c/\Delta v\) where \(c\) is the speed
of light. However, temperature variations and vibrations in the cavity decrease the practical value of $l_c^4$.

Before moving on, it is necessary to give a general definition of coherence. Typically, a coherent beam is one in which the light waves in the beam maintain a constant phase difference. However, in reality no beam of light stays completely coherent for an infinite period of time. This means that for some finite time period the light waves in the beam remain more or less in phase. At some point, however, the phases start to stray from their initial values and the coherence of the beam is lost. This characteristic time period is called the “coherence time” ($\tau_c$) of the beam. Logically, the “coherence length” ($l_c$) is then given by $l_c = c\tau_c$.

When referring to coherence, it is often useful to define a correlation function $f(\tau)$ as follows:

$$f(\tau) = \int g(t)g(t + \tau)dt$$  \hspace{1cm} (1)

where $g(t)$ is a periodic function such as $g(t) = \sin(\omega t + \phi)$. Figure 1 below illustrates the situation nicely.
For a completely coherent wave, \( g(t) = g(t + \tau) \) and the integral in equation (1) reduces to \( \int g(t)^2 dt \) which will, of course, be positive. However, as the phase changes and \( \tau \) approaches \( \tau_c \), the integral goes to zero since \( g(t) \) and \( g(t + \tau) \) become uncorrelated.

What we find is that the correlation function \( f(t) \) decays exponentially with time and that the characteristic decay time of this curve is the coherence time of the beam.

In their famous experiment in 1956, Hanbury Brown and Twiss determined the correlation between intensity fluctuations in light emitted from an Hg arc. The setup for their experiment is shown in Figure 2 below.

They then altered the degree of coherence between the beams by varying \( h \) and determined the correlation between intensity fluctuations at each point. Their results confirmed that the coherence of a beam of light is closely linked to intensity fluctuations within the beam. In our photon counting experiment, this fact essentially gives rise to the differing distributions for timescales longer and shorter than \( \tau_c \).
Naturally, for photon counting experiments involving monochromatic light sources we expect the results to obey some type of statistical distribution. For a laser on time scales longer than the coherence time, the intensity is constant. More specifically, over these “long” time periods the random fluctuations in intensity tend to cancel each other out\(^2\). The laser with its constant intensity is analogous to a radioactive source with a constant rate of decay. Both cases must obey a Poisson distribution. That is, the probability \(P(n)\) of observing \(n\) events, where \(<n>\) is the average, is given by

\[
P(n) = <n>^n e^{-<n>} / n!
\]

In the case of the laser, the intensity is logically given by

\[
I = <n>h\nu / tA
\]

where \(<n>/t\) is the average number of photons per unit time, \(h\nu\) is the energy of each photon, and \(A\) is the area of the detector. Solving for \(<n>\), we get \(<n> = \alpha I\) where we let \(\alpha = tA/h\nu\). Substituting for \(<n>\) in equation (2) gives

\[
P(n) = (\alpha I)^n e^{-(\alpha I)/n!}
\]

This equation gives the predicted distribution for time scales longer than the coherence time in this experiment.

Next, we can generalize equation (4) for light sources of varying intensities. For a light source whose intensity obeys a given probability density function \(P(I)\), we can simply multiply equation (4) by \(P(I)\) and integrate over all intensities, giving

\[
P(n) = \int ((\alpha I)^n/n!)e^{-\alpha I}P(I)dI
\]

This equation is known as Mandel’s formula\(^5\).

For timescales less than \(\tau_c\), however, the photons are distributed quite differently. Due to an effect known as “photon bunching”, the probability of detecting a photon
doubles immediately after one has been detected. In other words, the presence of one photon increases the likelihood that another will be present. This effect is explained by Bose-Einstein statistics, which is applicable for the He-Ne laser due to its Gaussian irradiance distribution brought about by its lack of transverse modes.

To derive the Bose-Einstein distribution, we begin by noting that the photons must obey the Boltzmann distribution:

$$P(E) = C_1 e^{-C_2 E}$$  \hspace{1cm} (6)

where \( E \) is the energy of the photon and \( C_1 \) and \( C_2 \) are constants. However, since energy is proportional to intensity we can replace \( E \) by \( I \) in equation (6) simply by choosing new constants. We can then determine \( C_1 \) and \( C_2 \) by solving the equations \[ C_1 e^{-C_2 I} dI = 1 \] and \[ C_1 e^{-C_1 I} dI = <I> \] where \( <I> \) is the average intensity. Doing so gives \( C_1 = C_2 = 1/<I> \). Substituting these values into equation (6) and plugging \( P(I) \) into Mandel’s formula gives, after some simplification,

$$P(n) = (\alpha)^n/(1+\alpha)^{n+1}. \hspace{1cm} (10)$$

This is the Bose-Einstein distribution with a mean of \( <I> \alpha \). This derivation is made possible by assuming that the photons are completely indistinguishable from each other, which is appropriate due to photons’ characteristics as bosons. Physically, for times less than \( \tau_c \) the time interval is short enough so that the random fluctuations in intensity can be observed directly, which results in a Bose-Einstein distribution.

For an Argon laser, the situation is significantly more complex due to the many longitudinal and transverse modes. The derivation of the Bose-Einstein distribution above does not apply to the argon laser due to its non-Gaussian irradiance distribution. Furthermore, the beam is not completely spatially coherent due to phase shifts in the
electric field within the beam\(^4\). Hence, while the Argon laser should obey a Poisson distribution for times longer than \(\tau_c\), its distribution for times shorter than \(\tau_c\) will not obey a Bose-Einstein distribution. These same predictions apply to each of the components of the Argon laser, separated by a prism.

Since \(\tau_c\) is on the order of 1 ns, it is easier to simulate this timescale than it is to actually measure it. This simulation is done by placing a spinning ground glass disk in the path of the beam. Due to irregularities sandblasted into the surface of the glass, different parts of the beam have different path lengths through the glass. Figure 3 below shows the light as it emerges from the disk.

![Diagram of the ground glass disk and laser beam](image)

The components of the beam have random amplitudes and phases as they emerge from the glass\(^5\). This results in a random interference pattern, termed a speckle pattern, at the
detector. The random intensity fluctuations created in the speckle pattern precisely simulate the intensity fluctuations of a laser on timescales shorter than \( \tau_c \). Furthermore, the rate of fluctuation can be controlled simply by adjusting the speed of the spinning glass disk. The simulated coherence time is the time it takes for one “grain” in the glass surface to pass through the beam, which corresponds to one specific speckle pattern at the detector.

The “noise” in this experiment comes from two main sources. The first source of noise are the so called “dark counts” which are caused by electrons thermally emitted by the photocathode of the PMT\(^5\). These electrons are indistinguishable from electrons emitted by the photoelectric effect that correspond to actual photon counts. The second source of noise is shot noise, which results from the fluctuating rate of photon counts\(^5\). This signal-to-noise ratio goes as \( \sqrt{<K>} \) where \( <K> \) is the average number of counts\(^5\). Hence shot noise is more significant for lower values of \( <K> \). However, as we will see, both sources of noise were negligible for most trials due to the short sampling times involved.

The final part of the experiment involves building a Michelson interferometer, shown below in Figure 4.
For two beams of intensity $I_0$, the intensity at the detector will obey the interference equation $I = 4I_0 \cos^2(\phi/2)$, where $\phi$ is the phase difference$^3$. Hence by vibrating one mirror over a distance greater than $\lambda/2$ using a Piezo motor, the intensity at the detector should vary from 0 to $4I_0$ for a coherent beam. For an incoherent beam, interference will not occur and the output should have a constant intensity of $2I_0$. Hence, by changing the path length difference between the beams and observing the output of the photodiode, one should be able to obtain an estimate of the laser’s coherence length.

**Experimental Method**

Figure 5 below shows the experimental setup for the first part of the experiment.
The setup for the experiment was relatively straightforward. We mounted the He-Ne laser to one end of the breadboard and passed the beam through a polarizer and an adjustable neutral density filter. Since the He-Ne laser beam is polarized to begin with, we were able to adjust the intensity of the beam at the detector by rotating the polarizer. Although this would ideally decrease the intensity to zero at right angles, we found that the polarizer could only attenuate the beam to a given minimum intensity. Since our experiment required extremely small photon counts, we used a neutral density filter to further attenuate the beam.

Next the beam passed through a lens placed precisely one focal length from the far edge of the glass disk. This focused the beam on the irregular surface of the glass and gave us the best possible speckle pattern. The disk was mounted (by the machine shop) to a variable speed dc motor that was powered using an adjustable dc power supply. Of course, for the Poisson trials the lens and glass disk were taken out of the path of the beam.

The beam then passed through two pieces of cardboard with pinholes inside a cardboard box. The PMT was placed at the back of the box behind the second pinhole.
The box and pinholes helped ensure that any light reaching the PMT came from the laser and not some other source.

The PMT we used was not a standard photomultiplier tube. Rather, it was specifically designed for low photon counts and gave off a brief 5V pulse for each photon it detected. The pulses then passed to a personal computer via an ADC card.

Next the pc read the pulses using a C program we wrote and the computer’s internal gated timer. The program allowed the sampling time to be set anywhere between .01 ms and 1 s. For each time interval, the program counted the number of HI pulses it received and stored these values in an array. These values were then sorted and placed into a histogram. After enough data had been taken, the histogram could then be saved directly into a Microsoft Excel file for further analysis.

Before taking any data with the spinning glass disk, we constructed a simple phototransistor circuit and placed it directly behind a pinhole in a piece of cardboard. We then spun the disk at its slowest possible speed and viewed the output of the phototransistor on an oscilloscope. This gave us an estimate for the simulated coherence time created by the spinning glass disk. We found this coherence time to be on the order of 1 ms. Hence choosing a sampling time of .01 ms satisfies the condition $\tau << \tau_c$ for the Bose-Einstein trials. Since $\tau_c$ was on the order of 1 ns without the disk, the .01 ms sampling time also satisfied the condition $\tau >> \tau_c$ in this case. For this reason, we usually used a .01 ms sampling time regardless of the type of trial being made.

This sampling time also helped minimize the effects of dark counts on our data. By running several data trials with the laser off, we determined the dark count rate to be in the range of 170-440 counts/sec. The exact dark count rate at any give moment was a
function of how long the voltage for the PMT had been left on. Using a thermocouple, we confirmed that the temperature of the PMT rises with time, which explains the rising dark count rate. At around 440 counts/sec, the PMT reaches thermal equilibrium and the rate levels off. Since the dark count rate was continuously changing, we turned the laser off after each data trial and quickly determined the dark count rate at that time. However, the exact rate is quite negligible since even a dark count rate of 440 counts/sec corresponds to .0044 counts per .01 ms interval.

In performing our data trials, we began by shutting off the room lights and adjusting the polarizer and density filters to give the desired average count rate. For each case, we took data for averages of roughly 1, 5, and 15 counts per time interval. These averages were chosen because they each give substantially different Poisson and Bose-Einstein distributions. When the desired average rate was obtained, we turned the computer screen off and began taking data. This helped to minimize the amount of stray light in the room. The exact number of .01 ms time intervals we measured varied somewhat for each trial, although it was usually in the hundreds of thousands of data points. This was sufficient to build up a smoothly shaped histogram.

After completing our trials with the He-Ne laser we repeated the trials in the same way using an Argon laser. Finally, we sent the Argon beam through a prism and repeated the trials on the 514 nm green emission line.

The second part of the experiment involves building the Michelson interferometer as shown in Figure 4. After its construction we plan to vary the path length difference using a stepper motor and observe the output of the photodiode to determine the point at which interference stops occurring. This path length difference will give us and estimate
for the coherence length of the beam. Since we know that $\tau_c$ is on the order of 1ns, this length should be roughly $c\tau_c$, or .3m.

**Preliminary Data**

The graphs at the end of this report show our data for several trials with and without the glass disk. As expected, the He-Ne laser trials agree quite well with the predicted Poisson and Bose-Einstein distributions. The Argon trials (with and without the prism) also agree quite well with the Poisson distribution for timescales longer than $\tau_c$. Finally, as expected the Argon trials do not obey a Bose-Einstein distribution due to the presence of transverse modes and the non-Gaussian irradiance field. Clearly, these results will be further analyzed for the final report.

**References**


