You may find the following equations helpful:

\[
\begin{align*}
\bar{v} &= \sqrt{\frac{8k_B T}{\pi m}} \\
\lambda &= \frac{k_B T}{\pi d^2 p} \\
\frac{Q_c}{A} &= K_c p (T_s - T_e) \\
\frac{Q_r}{A} &= \sigma (T_s^4 - T_e^4) \\
k_B &= 1.38 \times 10^{-23} \text{ kg} \cdot \text{m}^2/\text{K} \cdot \text{s}^2 \\
&= 8314 \text{ amu} \cdot \text{m}^2/\text{K} \cdot \text{s}^2
\end{align*}
\]

1. Consider the Pirani gauge sketched above. The distance from the hot wire to the enclosure is 0.5 cm. The wire had a radius of 0.3 mm and is 3 cm long. The coefficient of gas conductive heat transport is \(K_c = 1.2 \times 10^{-2} \text{ W/cm}^2 \cdot \text{K} \cdot \text{Pa}\) and the coefficient of radiative heat transport is \(\sigma = 5.7 \times 10^{-12} \text{ W/cm}^2 \cdot \text{K}^4\).

a) What is the approximate high-pressure limit for this gauge (at room temperature \(T_e = 300\text{ K}\)), assuming the diameter of an air molecule to be 0.3 nm?
(question continued from page 1)

b) Suppose we operate the gauge in constant temperature mode with $T_s = 350$ K. At what pressure will the heat loss by radiation be 50% of the heat loss by conduction? This is roughly the low-pressure limit of the gauge.

c) Suppose we operate the gauge in constant current mode with $I = 100$ mA and $R = \Lambda T_s = 15$ m$\Omega$/K$ \cdot T_s$. What is the relationship between the resistance measured and the pressure, assuming the heat loss to radiation to be negligible?
2. Given a turbopump with a rotor radius of 7 cm, what speed must the rotor spin at to have an velocity equal to the average velocity for nitrogen molecules at room temperature? How about for hydrogen molecules?

3. In an ion gauge, an electron travels through the volume, ionizing the gas. The measurement of the gas density is based on ion current measured.
   a) Derive the relationship between the average path length of a electron in an ion gauge and the number of ions produced at a given number density (pressure). There will be an arbitrary scale \( \zeta \) which encapsulates the efficiency of ion production with units of \( \text{cm}^2 \).
   
   b) Based on the result above, what is the relationship between the injected electron current \( I_e \), the measured ion current \( I_i \), and the number density?