1) **(15 Points)** In a measurement of absorption in a semiconductor, the amount of light absorbed increases exponentially over a very small wavelength range. It is sometimes necessary to measure a signal that varies over four or five orders of magnitude. This is very difficult to do with an ordinary amplifier. In this problem, we consider the following alternative:

You can assume that the op-amp is ideal. The purpose of the resistor R is to ensure that the feedback diode D is always in its conducting state. The photo-diode PD produces a current I_s that is proportional to the intensity of the incident light. The current I_s flows in the reverse direction as shown in the drawing.

a) Explain why all of the bias current I_B flows through the diode D. Indicate clearly how the assumptions of ideal op-amp behavior lead you to this conclusion.

b) Sketch the function \( I = I_0(e^{\alpha V} - 1) \) and explain in a few sentences why it is a reasonably good approximation for the I-V curve of a typical diode. In terms of \( \alpha \) and \( I_0 \), what are the reverse current and forward voltage drop (i.e. “turn-on voltage”) of the diode?

c) Assuming the I-V relation of part b), compute the output voltage \( V_{out} \) of this circuit for a given I_B and I_s. In a few sentences, explain why this circuit is a good choice for measuring light signals that vary over many orders of magnitude.

d) Alas, your light signal is very small, and you have to worry about the noise in your circuit. You can assume that the diode D and the battery are noiseless. On the other hand, the manufacturer of the op-amp specifies an input noise current of \( 2 \times 10^{-15} \text{ A/Hz}^{1/2} \). The manufacturer of the photodiode specifies an equivalent noise current of \( 2 \times 10^{-15} \text{ A/Hz}^{1/2} \) for the photodiode “in the dark”, i.e. when there is no light shining on it. The bias resistor R is
Given these three possible sources of noise, compute the rms noise output voltage of the circuit in the dark, assuming that you are running the output through a flat-top band-pass filter with a low-frequency cut-off of 2.0 kHz and a high-frequency cut-off of 4.0 kHz. You can assume that \( I_0 = 10^{-12} \text{ A} \) and that \( \alpha = 15 \text{ V}^{-1} \). Boltzmann’s constant is \( 1.38 \times 10^{-23} \text{ J/K} \).

e) You now turn on a light source, producing a photocurrent \( I_s \). At what light intensity will the shot noise due to the photocurrent be equal to the noise you calculated in part d)? The manufacturer specifies a sensitivity of 0.2 A/W, meaning that 1 watt of light produces a photocurrent \( I_s \) of 0.2 A.

2) (10 Points) You are in charge of a diplomatic conference that is attempting to unite seven rebellious provinces into a single nation. Of course the representatives of each province are very jealous of each other and are always fighting over the order in which they should speak. After some thought you decide on the following scheme:

A table is created with four columns, labeled A, B, C, and D. The first rule that you set is that the entry for column A in any row will be 1 if either C or D, but not both, are equal to 1. Otherwise, the entry in column A for that row is zero. In the first row of the table you make B, C, and D equal to 1. For subsequent rows, you decide that the entries for columns B, C, and D will be taken from columns A, B, and C of the previous row, in that order. Your rule for preserving peace at the conference is very simple: The provinces each receive a number from 1 to 7. At the beginning of each hour, the entries of columns B, C, and D will be converted to a decimal number, with B the most significant bit and D the least significant bit. This number determines which province gets to speak. At the beginning of each hour, you move one row down the table.

a) Write down 8 rows of this table, starting with \( B = C = D = 1 \). Explain in a few sentences why everyone agrees that you have arrived at a very fair scheme for determining the speaker order.

\[
\begin{array}{cccc}
A & B & C & D \\
1 & 1 & 1 & \\
&&&
\end{array}
\]

b) Your scheme is so successful that you decide to automate it. Using only components that you have used in 4051 laboratory, design a circuit that will light up three diodes corresponding to the correct values of B, C, and D at the beginning of each hour. You can assume that you have a TTL square wave generator available with a period of one hour. Include the diodes in your circuit.
3) (10 Points) A C-program uses an ADC card with an input voltage range of -10V to +10V to acquire N data points from a time varying voltage source. It converts each data point to the appropriate voltage and then stores them consecutively in a local data array of type double.

It is your job to write a C-function that can be called after the data acquisition has been completed, which:
- Returns the minimum voltage, i.e., the value of the lowest voltage acquired and stored in the array;
- Reverses the order in which the voltages are stored in the array, i.e., the first voltage reading will be stored in the last element, the second reading in the second to the last element, etc.

The function must have two (and only two) arguments: one is a pointer to the array containing the data and the other specifies the number of array elements.

You should use appropriate data types for the arguments and the variables in the function. You may name the function anything except "main." Write the entire function with function declaration, header, body and return statement. You may write it using as many or as few additional local variables as you consider necessary.

Note: You do NOT need to check for the special case where N < 2. Also, if you have difficulty visualizing the function, you may break the function into two separate, distinct functions: the first one returns the lowest value in the array and the second one reverses the elements in the array.

You will be graded on a) program logic and b) number of syntax mistakes.

5) (9 Points) The AC signal \( y = \cos(2\pi ft) \) is half-rectified as shown in the figure. (The actual signal continues \( ad infinitum \).) Compute the Fourier series expansion of this signal. Note the trigonometric identities:

\[
\sin(A + B) + \sin(A - B) = 2\sin A \cos B \\
\cos(A + B) + \cos(A - B) = 2\cos A \cos B
\]
6) (6 Points) Parts a) and b) pertain to the square wave shown in the figure below:

![Square Wave](image)

**a)** You obtain an FFT of the square wave. You sample 2048 equally spaced points collected over 2.048 seconds. Sketch the FFT amplitude spectrum as quantitatively as possible, showing only the peaks that are at least 10% of the size of the largest peak.

**b)** For the same square wave, you collect the 2048 points in 4.55 seconds. Sketch the FFT amplitude spectrum, showing all peaks that are at least 10% of the size of the largest peak.
Problem 3:
The code in bold print is the solution to problem 3.

#include <ansi_c.h>
double FMin( double *ar, short pts);
#define MAX 10

main()
{
    double vin[MAX], min;
    short i;

    for(i = 0; i < MAX; i++)
    {
        vin[i] = 10.0 *rand()/RAND_MAX;
        printf("%d\t%.lf\n", i, vin[i]);
    }

    min = FMin( vin, MAX);
    printf("Minimum: %.lf\n", min);

    for(i = 0; i < MAX; i++)
        printf("%d\t%.lf\n", i, vin[i]);
}

double FMin( double *ar, short pts)
{
    double tmin = +10.0, temp;
    int i;

    for( i = 0; i < pts/2; i++)
    {
        temp = ar[i];
        ar[i] = ar[pts - i - 1];
        ar[pts - i - 1] = temp;
    }

    for( i = 0; i < pts; i++)
        if( ar[i] < tmin ) // find min
            tmin = ar[i];

    return( tmin);
}